# A possibility to solve the problems with quantizing gravity

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## Abstract

It is generally believed that quantum gravity is necessary to resolve the known tensions between general relativity and the quantum field theories of the standard model. Since perturbatively quantized gravity is non-renormalizable, the problem how to unify all interactions in a common framework has been open since the 1930s. Here, I propose a possibility to circumvent the known problems with quantizing gravity, as well as the known problems with leaving it unquantized: By changing the prescription for second quantization, a perturbative quantization of gravity is sufficient as an effective theory because matter becomes classical before the perturbative expansion breaks down. This is achieved by considering the vanishing commutator between a field and its conjugated momentum as a symmetry that is broken at low temperatures, and by this generates the quantum phase that we currently live in, while at high temperatures Planck's constant goes to zero.

Keywords: Quantum gravity, Semi-classical gravity

## 1. Why Quantize Gravity?

The gravitational interaction stands apart from the other interactions of the standard model by its refusal to be quantized. This still missing theory of quantum gravity is believed necessary to complete our understanding of nature. Strictly speaking, quantizing gravity is not the problem – gravity can be perturbatively quantized. The problem is that the so quantized theory is perturbatively non-renormalizable and cannot be understood as a fundamental theory. It breaks down at high energies when quantum gravity would be most interesting.

The attempt to find a theory of quantum gravity has lead to many proposals, but progress has been slow. Absent experimental evidence, reasons for the necessity of quantum gravity are theoretical, most notably:

- 1. Classical general relativity predicts the formation of singularities, infinite energy densities, under quite general circumstances. Such singularities are unphysical and should not occur in a fundamentally meaningful theory.
- 2. Quantum field theory in a curved background leads to black hole evaporation. Black hole evaporation however seems to violate unitary which is incompatible with quantum mechanics. It is widely believed that quantum gravitational effects restore unitarity and information is conserved.
- 3. There is no known consistent way to couple a quantum field to a classical field, and since all quantum fields carry energy they all need to couple to the gravitational field. As Hannah and Eppley have argued [1], the attempt to make such a coupling leads either to a violation of the uncertainty principle (and thus would necessitate a change of the quantum theory) or to the possibility of superluminal signaling, which brings more problems than it solves. Mattingly has argued [2] that Hannah and Eppley's thought experiment can't be carried out in our universe, but that doesn't solve the problem of consistency.

These points have all been extensively studied and discussed in the literature. The most obvious way out seems to be a non-perturbative theory, and several attempts to construct one are under way.

It is worthwhile for the following to identify the problems with coupling a classical to a quantum field.

The one problem, as illuminated by Hannah and Eppley is that the fields would have different uncertainty relations, and their coupling would require a modification of the quantum theory. Just coupling them as they are leads to an inconsistent theory. The beauty of Hannah and Eppley's tought argument is its generality, but that is also its shortcoming, because it does not tell us how a suitable modification of quantum theory could allow such a coupling to be consistent.

The second problem is that it is unclear how, mathematically, the coupling should be realized, as the quantum field is operator-valued and the classical field is a function on space-time. One possible answer to this is that any function can be identified with an operator on the Hilbert space by multiplying it with the identity. However, the associated operators would always be commuting, so they are of limited use to construct a geometrical quantity that can be set equal to the operator of the stress-energy-tensor (SET) of the quantum fields.

Another way to realize the coupling is to extract a classical field from the operator of the SET by taking the expectation value. The problem with this approach is that the expectation value may differ before and after measurement, which then conflicts with local conservation laws in general relativity. Coupling the classical field to the SET's expectation value is thus usually considered valid only in the approximation when superpositions carry negligible amounts of energy.

These difficulties can be circumvented by changing the quantization condition in such a way that gravity can be perturbatively quantized at low energies, but at energies above the Planck energy – energies so high that the perturbative expansion would break down – it becomes classical and decouples from the matter fields. The mechanism for this is making Planck's constant into a field that undergoes symmetry breaking and induces a transition from classical to quantum. In three dimensions, Newton's constant is  $G = \hbar c/m_{\text{Pl}}^2$ , so if we keep mass units fix, G will go to zero together with  $\hbar$ , thus decoupling gravity.

It should be emphasized that the ansatz proposed here does not renormalize perturbatively quantized gravity, but rather replaces it with a different theory that however reproduces the perturbative quantization at low energies by construction. We will show in the following how this change of the quantization condition addresses the above listed three problems.

In the approach considered here, Planck's constant is treated as a field. Since the normal, fixed, value of Planck's constant (denoted  $\hbar_0$ ) appears as a vacuum expectation value, we could divide the field by this vacuum expectation value to obtain a dimensionless quantity. We will not do this in the following, because it would make the interpretation less intuitive. It should be pointed out though that the approach can be reformulated in terms of a dimensionless field because we do have access to a dimensionful constant that serves as reference.

In the following we set c = 1 but keep  $\hbar$  and Boltzmann's constant  $k_B$ . The signature of the metric is (+, -, -, -).

#### 2. Quantization by Spontaneous Symmetry Breaking

Consider a massless real scalar field  $\phi(x,t)$  with canonically conjugated momentum  $\pi_{\phi}(x,t)$ . Second quantization can be expressed through the equal time canonical commutation relations

$$[\phi(x,t),\pi_{\phi}(y,t)] = i\hbar\delta^3(x-y) \quad , \tag{1}$$

or, equivalently, by the commutation relations for annihilation and creation operators. If  $\hbar = 0$ , they commute and one is dealing with a classical field.

The dimension of a scalar field is most easily found by noting that the kinetic term  $\partial_{\nu}\phi\partial^{\nu}\phi$  in the Lagrangian should have dimension of an energy density, so that

the integral over space-time has the dimension of an action. One has a freedom here whether a constant is in front of this term. If one is dealing with a quantum theory, one often puts an  $\hbar^2$  there because then each derivative together with an  $\hbar$  gives a momentum. Since we eventually want to make contact to a classical theory, we will not put any  $\hbar$ 's in front of the kinetic term, but instead chose the classical convention from the start on.

In three spatial dimensions this mean then that the dimension of the Lagrangian is  $[E/L^3]$ , where *E* denotes energy and *L* denotes length. So the dimension of  $\phi$  is  $[E^{1/2}L^{-1/2}]$ , and that of the conjugated momentum  $\pi_{\phi}$  is  $[E^{1/2}L^{-3/2}]$ . The Lagrangian is

$$\mathcal{L} = \frac{1}{2} \partial^{\nu} \phi \partial_{\nu} \phi \quad , \tag{2}$$

without additional factors, and  $\pi_{\phi} = \partial \mathcal{L} / \partial \dot{\phi}$  as usual. With this convention, what is in the quantum theory referred to as the 'mass term' has actually the form  $\phi^2 / l_*^2$ , where  $l_*$  is a length scale. This is because the unquantized field is not a priori associated with particles that could be assigned a mass.

With that dimensional consideration, let us now look at the quantization prescription from a new perspective. The requirement that the fields commute,  $\phi(x)\pi(y) = \pi(y)\phi(x)$ , can be understood as a symmetry, where we find the classical theory as the symmetric, commuting, phase and quantum mechanics in the phase with broken symmetry where the fields do not commute.

In the familiar language of spontaneous symmetry breaking we parameterize

$$[\phi(x,t),\pi_{\phi}(y,t)] = \mathrm{i}\frac{\hbar_0^2}{m_*}\alpha(x,t)\delta^3(x-y) \quad . \tag{3}$$

Here,  $\hbar_0$  is the normal (measured) value of Planck's constant,  $m_*$  is a constant of dimension mass, and  $\alpha(x,t)$  is a real scalar field. With a Fourier-transform, one can express this in terms of annihilation and creation operators for  $\phi$  of the form

$$[a_{\vec{k}}, a_{\vec{k}'}^{\dagger}] = \mathrm{i} \frac{\hbar_0^2}{m_*} \left( \frac{\omega_{k'} + \omega_k}{2\sqrt{\omega_{k'}\omega_k}} \right) \tilde{\alpha}(\vec{k} - \vec{k}') \tag{4}$$

where  $(\omega_k, \vec{k})$  is the wavevector,  $\omega_k^2 = |\vec{k}|^2$ , and  $\tilde{\alpha}$  is the Fourier transform of  $\alpha$ 

$$\tilde{\alpha}(\vec{k}) = \int d^3x \, \alpha(x,t) \exp(ix \cdot k) \quad .$$
(5)

If  $\alpha$  is constant, then  $\tilde{\alpha}$  reduces to the usual delta-function. For the operations leading to the above expressions, we have merely used the definition of scalar products that are not affected by the modification of the quantization condition.

We have distributed the dimensionful constant so that  $\alpha$  has no prefactor in the path integral, ie the dimension of  $\alpha$  is [1/L]. We have further chosen to keep a mass scale fixed rather than a length scale for the reason indicated in the introduction. We can then write  $\hbar(x,t) = \hbar_0^2 \alpha/m_*$  for Planck's constant, which is now a field. The field  $\alpha$  itself is quantized according to the same prescription as  $\phi$  up to constants, ie  $[\alpha(x,t), \pi_{\alpha}(y,t)] = i\hbar_0\alpha(x,t)\delta^3(x-y)$ . If we expand  $\alpha$  in annihilation and creation operators, these too obey the commutation relation (5) with a different prefactor.

Now we add a kinetic term for  $\alpha$  and a symmetry breaking potential, so that the transition amplitude in the path integral is  $\exp(-i\hat{S})$ , where the hat indicates a dimensionless quantity, and

$$\hat{S} = \int d^4 x \sqrt{-g} \left( \frac{m_*}{2\alpha\hbar_0^2} \partial^{\nu} \phi \partial_{\nu} \phi + \frac{1}{2} \partial^{\nu} \alpha \partial_{\nu} \alpha - V(\alpha) / \hbar_0 \right)$$

$$V(\alpha) = -2 \frac{m_*^2}{\hbar_0} \alpha^2(x) + \hbar_0 \alpha^4(x) \quad . \tag{6}$$

The minima of the potential are at  $\alpha = \pm m_*/h_0$ , and we live in the vacuum with  $\hbar = \hbar_0$ . (In order to render the kinetic term of  $\phi$  symmetric under a sign change of  $\alpha$ , one would have to also change the sign of  $m_*$  along with  $\alpha$ . This makes the example of the scalar field somewhat unappealing, but we will see below that this is not necessary for the case of gravity, which is what we are actually interested in.)

As mentioned in the introduction, to obtain a dimensionless quantity whose value is meaningful regardless of units, one could now normalize the field  $\hbar$  to  $\hbar_0$ . However, this would make the interpretation of the appearing quantities less intuitive, so we will not do this redefinition here. The previously made statement that the failure of the field and its momentum to commute represents a breaking of symmetry should be understood as a rephrasing of the breaking of symmetry in the ground state of the above potential.

This whole exercise can be summarized by saying that we've chosen the dimensions of the fields so that in the path integral the field  $\alpha$ , which determines Planck's constant, appears in the usual form.

Thus, we are left with an action of two scalar fields, where the symmetry breaking part works as normally. The only unusual is the quantization condition for the fields. Symmetry breaking then happens with a drop of temperature because the minima of the potential change with the temperature [3]. In the limit  $T \gg T_c = 2m_*k_B$ , the finite temperature corrected potential receives additional terms

$$V(T,\alpha) = V(\alpha) + \frac{1}{2\hbar_0} (k_{\rm B}T)^2 \alpha^2 - \frac{\pi^2}{90} nk_{\rm B}T + \dots \quad , \tag{7}$$

where  $k_{\rm B}$  is Boltzmann's constant, *n* is the number-density of the field, and the dots indicate terms of higher order in  $\hbar_0$ . Note that they are indeed higher order in  $\hbar_0$  and not in  $\hbar$  because there's no additional  $\hbar$  in the potential. The first correction term can be interpreted as a temperature-dependent mass term for the  $\alpha$ -field that counteracts the negative mass term in the potential. The second correction term is the free energy of a massless spin-0 boson.

To understand what happens if the symmetry is broken, we first note that in principle the temperature dependence of the potential is not a quantum effect, it is an in-medium effect. However, Planck's constant appears in statistical mechanics as the normalization constant for the measure of momentum space. Taking it to zero does not create a classical limit, but instead ill-defined quantities. One frequently considers limits in which Planck's constant is small compared to some other quantity of dimension action. That is formally similar to changing Planck's constant, but has a very different physical meaning. We don't want to look at the limit where Planck's constant is small compared to some other value, we want to change it. This is not a process that statistical thermodynamics in its standard form deals with.

Eg the free energy of the bosonic gas that appears in the potential is normally written  $(kT)^4/\hbar^3$ , which seems to diverge with  $\hbar$  to zero. But to obtain a meaningful limit, we have to keep a physically meaningful quantity fixed. As the above notation already suggests, we opt to keep the number density fixed when varying  $\alpha$  but not any other variable, ie

$$\frac{\partial n}{\partial \alpha}\Big|_{(T,S,p)} = 0 \quad . \tag{8}$$

This does of course not mean the number density is constant. To begin with it depends on the temperature. What Eq (8) expresses is that there is no additional temperature-dependence in *n* from the variation of  $\alpha$ . In other words, we identify  $n = (kT/\hbar_0)^3$ .

There is another way to look at this limit. Planck's constant is the measure in phase space, and can be understood as a product of a length for the coordinate space and an energy for momentum space. We have introduced a mass scale in the beginning, which we keep fix, so then the unit for the length has to go to zero with Planck's constant, which is why the number-density would diverge in these units. This unphysical behavior is cured by choosing a constant unit of length instead, which has the same effect as putting  $\hbar_0$  in the denominator of *n*.

The potential for the quantized scalar field further receives corrections from quantum fluctuations which leads to the Coleman-Weinberg effective potential. In the usual case for a  $\phi^4$ -interaction, the loop corrections become large for small  $\phi$  and therefore quantum fluctuations can break the classical symmetry. Since in our case the potential for the field  $\alpha$  takes the usual form, the same can happen here. It is less clear what happens with the loop corrections in the high-temperature limit when the modification of the quantization condition becomes relevant.

As mentioned above, what we actually want to unquantize is not a  $\phi^4$  theory, but gravity. In this case, the normalized action is

$$\hat{S} = \int \mathrm{d}^4 x \sqrt{-g} \left( \frac{m_{\rm Pl}^2 m_*^3}{\alpha^2 \hbar_0^4} \mathcal{R} + \frac{1}{2} \partial^{\rm v} \alpha \partial_{\rm v} \alpha - V(\alpha) / \hbar_0 \right) \,. \tag{9}$$

Other matter fields can be added like the scalar field example. This looks very much like Brans-Dicke, but has a different kinetic term and the symmetry breaking potential. The most relevant difference though is the quantization condition. (Note that the first term does not change sign under  $\alpha \rightarrow -\alpha$ .)

In summary, for this to work, one needs the normalized action of the form (6) together with the quantization postulate (3). Since the quantization postulate is essentially a constraint on the fields, one could alternatively add it with a Lagrange multiplier to the Lagrangian. That might not be particularly elegant, but the point is here just to show that it can be done.

#### 3. How unquantization addresses the problems with quantizing gravity

The coupling constant of gravity to matter in the perturbative quantization is  $\sqrt{G} = \sqrt{\hbar}/m_{\text{Pl}}$  and thus goes to zero with  $\hbar \rightarrow 0$ , provided the Planck mass is held fix. This limit was previously discussed from a different perspective in [4, 5] as being of interest as a non-quantum relic of quantum gravity in the limit  $\hbar, G \rightarrow 0$ . Here, we suggested to not only consider this a corner of the parameter space, but a limit that is actually realized at high energies.

The weakening of gravity at high energies can also be found in scenarios where gravity is asymptotically safe. The difference between both cases is most easily shown by a figure, see Fig 1.



Figure 1: The parameter space for Newton's constant G and Planck's constant  $\hbar$ , normalized to the measured values  $G_0$  and  $\hbar_0$ . The black cross indicates the familiar lowenergy theory, the place where quantization of general relativity is perturbatively nonrenormalizable. Shown in grey are the hyperbolas where the Planck length is constant, and the straight lines where Planck's mass is constant. In Asymptotically Safe Gravity (ASG), Newton's constant decreases at high energies, weakening gravity. In the here discussed scenario of unquantization, Planck's constant goes also to zero.

Let us now revisit the motivations for the need for quantum gravity:

- 1. The formation of singularities: If matter is compressed, it eventually forms a degenerate Fermi gas. If it collapses to a black hole, it collapses rapidly and after horizon formation lightcones topple inward, so no heat exchange with the environment can take place and the process is adiabatic. The entropy of the degenerate Fermi gas is proportional to  $Tn^{-2/3}$ , which means that if the number density rises and entropy remains constant, the temperature has to rise [6]. If the temperature rises, then gravity eventually decouples and there is no force left to drive the formation of singularities. Towards the Big Bang singularity, temperature also raises and the same conclusion applies. Note that the unquantizing field makes a contribution to the source term, necessary for energy conservation.
- 2. The black hole information loss: If there is no singularity, there is no information loss problem [7]. Moreover, if the matter inside the apparent horizon becomes classical and has no Pauli exclusion principle, there's nothing preventing a remnant from storing large amounts of information that can be very suddenly released once the black hole has evaporated enough.
- 3. The problem of coupling a classical to a quantum theory: There is never

a classical field coupled to a quantized field. There is a phase when both are quantized and coupled, and one when both are unquantized and decoupled. One might say that fundamentally the fields are neither classical nor quantum in the same sense that water is fundamentally neither liquid nor solid.

It remains to be addressed what happens to the divergence of the perturbative expansion of perturbatively quantized gravity. This is necessary to understand what happens in a highly energetic scattering event where the mean field approximation that is made use of for the symmetry restoration with temperature is not applicable.

We first note that at high energies the operator that contributes the negative mass term to the potential of  $\alpha$  becomes less relevant than the quartic term, so at least asymptotically the symmetry should be restored in the sense that the vacuum expectation value of  $\hbar$  goes to zero. With the field rescaling that we have used here (see also [8]), the perturbative expansion is also an expansion in  $\hbar$ , but it is not quite as easy as this for the following reason.

Consider an S-matrix expansion within the above model. The expansion of the S-matrix works as usual, but the modification comes into play when we look at a transition amplitude of that S-matrix with some interaction vertices. It is evaluated by using the commutation relations repeatedly until annihilation operators are shifted to the very right side, acting on the vacuum, which leaves *c*-numbers (or the Feynman rules respectively). Now, if Planck's constant is a field, every time we use the commutation relation, we get a power of  $\alpha$  and the respective factors of the constant  $h_0$  and  $m_*$ . So, in the end we do not have powers of the vacuum expectation value of  $\tilde{\alpha}$ , but the expectation value of powers of  $\tilde{\alpha}$ . We thus need to take a second step, that is using the commutation relations on  $\tilde{\alpha}$  itself. But exchanging any two terms in the expansion of  $\tilde{\alpha}$  will only generate one new  $\tilde{\alpha}$ from the commutator (and the respective powers of  $\hbar_0$ ). One can thus get rid of the expectation value of  $\tilde{\alpha}$ .

If  $\alpha$  goes to zero only for infinitely large energies, then one cannot tell if the series is finite without further investigation of its convergence properties. However, if we consider the symmetry breaking potential to be induced by quantum corrections at low order, the transition to full symmetry restoration may be at a finite value of energy. In this case then, the quantum corrections which would normally diverge would cleanly go to zero, removing this last problem with the perturbative quantization of gravity.

We have discussed here a model based on a modification of the quantization condition that we have interpreted as a procedure of unquantization. This modification should however primarily be understood as a motivation for the model, and as a guide for the interpretation of its effects. It is possible that the approach can be reformulated in other ways that offer a different interpretation of the behavior.

The solution proposed here has the potential to address a long standing problem in theoretical physics. To be successful in that however, a closer investigation is required. There is, most importantly, the question of experimental contraints from coupling the scalar field that is Planck's constant to gravity, which might lead to a modification of general relativity and observable consequences. It also remains to be seen if a concrete example for the symmetry breaking can be constructed in which it can be shown explicity, and beyond the general argument for such a possibility given above, that the perturbation series converges. The betafunction of the model and its relation to the case of Asymptotically Safe Gravity is of key interest here. And while the avoidance of the Big Bang and black hole singularities are the most relevant cases for our universe, it remains to be seen if a more widely applicable statement can be derived that addresses the singularity theorems in general.

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#### References

- [1] K. Eppley and E. Hannah, "*The Necessity of Quantizing the Gravitational Field*," Foundations of Physics, 7:5165, (1977).
- [2] J. Mattingly, "Why Eppley and Hannah's thought experiment fails," Phys. Rev. D 73, 064025 (2006) [gr-qc/0601127].
- [3] J. I. Kapusta, "Finite-temperature field theory," Cambridge University Press, Cambridge (1993).
- [4] J. Kowalski-Glikman, "Doubly special relativity: Facts and prospects," In Oriti, D. (ed.): Approaches to quantum gravity 493-508. [gr-qc/0603022].
- [5] L. Smolin, "Classical paradoxes of locality and their possible quantum resolutions in deformed special relativity," arXiv:1004.0664 [gr-qc].

- [6] D. S. Kothari, *Joule-Thomson effect and adiabatic change in degenerate gas*, Proc. Nat. Inst. Sci. India, Vol 4, p. 69 (1938).
- [7] S. Hossenfelder and L. Smolin, "*Conservative solutions to the black hole information problem*," Phys. Rev. D **81**, 064009 (2010) [arXiv:0901.3156 [gr-qc]].
- [8] S. J. Brodsky and P. Hoyer, "*The h̄ Expansion in Quantum Field Theory*," Phys. Rev. D 83, 045026 (2011) [arXiv:1009.2313 [hep-ph]].