Zero Cosmological Constant and Nonzero Dark Energy from Holographic Principle

Jae-Weon Lee*

Department of energy resources development, Jungwon University, 5 dongburi, Goesan-eup, Goesan-gun Chungbuk Korea 367-805 (Dated: December 16, 2018)

It is shown that the first law of thermodynamics and the holographic principle applied to an arbitrary large cosmic causal horizon naturally demand the zero cosmological constant and non-zero dynamical dark energy in the form of the holographic dark energy. Semiclassical analysis shows that the holographic dark energy has a parameter d = 1 and an equation of state comparable to current observational data, if the entropy of the horizon saturates the Bekenstein-Hawking bound. This result indicates that quantum field theory should be modified at large scale to explain dark energy. The relations among dark energy, quantum vacuum energy and entropic gravity are also discussed.

^{*} scikid@jwu.ac.kr

Type Ia supernova (SN Ia) observations [1, 2], the Sloan Digital Sky Survey (SDSS) [3–6] and cosmic microwave background observations [7] all indicate that the current universe is expanding at an accelerating rate. The expansion can be explained if there is a negative pressure fluid called dark energy of which pressure p_{DE} and energy density ρ_{DE} satisfy $w_{DE} \equiv p_{DE}/\rho_{DE} < -1/3$. Being one of the most important unsolved puzzles in modern physics and cosmology, the dark energy problem consists of three sub-problems [8]; why it is so small, nonzero, and comparable to the critical density at the present.

We also need to explain why the cosmological constant Λ is so small or exactly zero. Solving this problem is not an easy task, because quantum field theory (QFT) predicts huge zero point energy that can play a role of Λ . It is very hard to reconcile the great success of QFT at small scales with this failure of QFT to explain dark energy. There are already many works on this problem [9–11], however, the problem seems to be far from a solution.

In this paper, it is suggested that if the holographic principle holds for a cosmic causal horizon, the cosmological constant should be exactly zero and there should be holographic dark energy (HDE) consistent with the recent observational data. The holographic principle [12] is a conjecture claiming that all of the information in a region can be described by the physics at the boundary of the region and that the maximal number of degrees of freedom in the region is proportional to its surface area rather than the volume. More specifically, it was conjectured that the Bekenstein-Hawking Entropy

$$S_{BH} = \frac{c^3 A}{4G\hbar} \tag{1}$$

is the information bound that a region of space with a surface area A can contain [13].

Based on black hole physics Cohen et al [14] proposed that the total energy in a region can not be larger than that of a black hole of that size. Therefore, if the region has a size $r_h = O(H^{-1})$, the vacuum energy density is bounded as $\rho_{\Lambda} \leq O(M_P^2 H^2)$, where H = da/adt is the Hubble parameter with the scale factor a, and $M_P = \sqrt{\hbar c/8\pi G}$ is the reduced Planck mass. Interestingly, saturating the bound gives HDE comparable to the observed dark energy density $\rho_{\Lambda} \sim 10^{-10} eV^4$. However, Hsu [15] pointed out that with the Hubble horizon HDE behaves like matter rather than dark energy. Li [16] suggested that using the future event horizon as IR cutoff we can solve this problem.

Recently, based on the holographic principle Verlinde [17] and Padmanabahan [18] proposed a remarkable idea linking gravity to entropy, which brings out many follow-up studies [19–30]. Verlinde derived the Newton's equation and the Einstein equation by assuming that energy inside a holographic screen is the equipartition energy $E_h \sim T_h N$ for the screen with the temperature T_h and the number of bits N.

On the other hand, in a series of works [31–34], Lee et al. suggested that the energy of gravitational systems could be explained by considering information loss at causal horizons. For example, we pointed out that a cosmic causal horizon with a radius $r \sim O(H^{-1})$ has temperature $T_h \sim 1/r$, entropy $S_h \sim r^2$ and a kind of thermal energy $E_h \sim T_h S_h \sim r$, which can be dark energy [35]. This dark energy, dubbed 'quantum informational dark energy' [36] or 'entanglement dark energy' [31] by the authors, is similar to the entropic dark energy based on the Verlinde's idea [37–40]. It was also suggested that black hole mass and the Einstein equation itself can be derived from the relation $dE_h = k_B T_h dS_h$, that might have a quantum information theoretic origin [32]. Similarities between this theory and Verlinde's theory were investigated in [34, 41].

In this paper we assume that the holographic principle and the following first-law like definition of the horizon energy

$$dE_h \equiv k_B T_h dS_h,\tag{2}$$

hold for a cosmic causal horizon such as the cosmic event horizon or the apparent horizon. This energy could be the equipartition energy [17], energy from Landauer's principle associated with information loss at the horizon [31] or simply the energy defined by the Clausius relation. Inspired by the entropic [17, 42] or quantum information theoretic [31, 34] interpretation of gravity we take the holographic principle and the horizon energy in Eq. (2) as guiding principles for dark energy study.

Let us first recall the cosmological constant problem in the context of QFT. The (classical) time independent cosmological constant Λ_c appears in the gravity action as

$$S = \int d^4x \sqrt{-g} (R - 2\Lambda_c). \tag{3}$$

Since the energy-momentum tensor $T_{\mu\nu}$ for the vacuum fluctuation $\langle T_{\mu\nu} \rangle$ is usually proportional to a spacetime metric (See for example [43]), $\langle T_{\mu\nu} \rangle$ has been regarded as a candidate for the cosmological constant and dark energy. To calculate its expectation value one usually integrates the zero point energy $\hbar\omega/2$ for each mode of quantum fields in a flat spacetime. Thus, the energy density of the quantum vacuum is approximately given by

$$\rho_q = \langle T_{00} \rangle \sim \int_{k_I}^{k_U} \hbar \omega d^3 k \sim k_U^4, \tag{4}$$

where $k_U \sim M_P$ is a UV-cutoff and $k_I \sim 1/r$ is an IR-cutoff. Unfortunately, as is well known, for $k_U \sim M_P$, the estimation gives $\rho_q \sim M_P^4 \sim 10^{109} eV^4$ which is too large to explain the observed dark energy density $\rho_{DE} \sim 10^{-12} eV^4$. On the other hand, if we subtract this zero point energy to calculate the renormalized vacuum energy for the universe, we obtain $\rho_q \sim H^4$, which is too small compared to the observed dark energy.

It is often argued that after taking the vacuum expectation of quantum fields, the Friedmann equation

$$H^{2} = \frac{8\pi G\rho_{m}}{3} - \frac{k_{c}c^{2}}{R} + \frac{\Lambda c^{2}}{3},$$
(5)

gets an additional constant contribution $\Lambda_q = \rho_q / M_P^2 c^2 = \langle T_{00} \rangle / M_P^2 c^2$ from the vacuum quantum fluctuation ρ_q in Eq. (4). (Here, k_c is the spatial curvature parameter, which we will set zero for simplicity, and ρ_m is the matter energy density.) Thus, the total cosmological constant is $\Lambda = \Lambda_c + \Lambda_q$, and the total vacuum energy density is given by

$$\rho_{vac} = M_P^2 c^2 (\Lambda_c + \Lambda_q). \tag{6}$$

Without a fine tuning it seems to be almost impossible for two terms to cancel each other to result in the tiny observed upper bound for the cosmological constant. This is the essence of the cosmological constant problem.

Then, in the context of QFT, from where could horizon energy ρ_h arise? Recall that ρ_q in Eq. (4) was estimated in a flat spacetime. However, for a curved spacetime, after a Bogoliubov transformation there appear excited states in addition to the vacuum. The normal ordered quantum vacuum energy (i.e., with the subtraction of the zero point energy) in a curved spacetime with the UV and the IR cutoffs often has a term in the form of ρ_h . One can do a volume integral of $\hbar\omega$ with the Bogoliubov coefficient β_k for the quantum fields in a curved spacetime to obtain ρ_h . For example, using the result in [47], it was shown in [46] that ρ_{vac} for the de Sitter universe contains an extra term

$$\rho_{vac}' \sim \int_{k_I}^{k_U} d^3k \ \hbar \omega |\beta_k|^2 \sim k_U^2 H^2 \tag{7}$$

in addition to the usual zero point energy, where $\beta_k \sim H/k$. If we choose M_P as the UV-cutoff k_U , this extra term gives $\rho'_{vac} \sim M_P^2 H^2 \sim \rho_h$. Thus, it is possible that ρ_h is actually the average quantum fluctuation energy above the zero point vacuum energy of the curved spacetime in the bulk [48]. This dark energy may be also identified to be the energy of cosmic Hawking radiation [46]. However, this calculation still can not explain why we can ignore the zero point energy in the bulk. It seems that there is no plausible way to overcome this difficulty, as long as we rely on the conventional QFT. We need another fundamental ingredient to solve this problem.



FIG. 1. A cosmological causal horizon Σ with a radius r, temperature T_h , and entropy S_h has a dark energy E given by $dE = k_B T_h dS_h \sim r$. The holographic principle for arbitrary r demands the cosmological constant to be exactly zero.

Alternatively, we can take not the bulk QFT but the holographic principle as a postulate and describe the bulk physics using only the DOF on the horizon. In this holographic context, to estimate the bulk energy density we can treat the quantum fields on the horizon as a collection of oscillators on the spherical surface with a lattice constant of order $O(k_U^{-1})$. Then, to obtain ρ_h we have to sum the zero-point energy of the oscillators with frequency ω on the horizon surface rather than those in the bulk. This rough estimation results in the HDE density, because

$$\rho_h \sim \Sigma_i \frac{\hbar\omega}{volume} \sim \Sigma_i \frac{\hbar\omega}{r^3} \sim \left(\frac{r}{k_U^{-1}}\right)^2 \frac{\hbar\omega}{r^3} \sim \frac{M_P^2}{r^2} \sim M_P^2 H^2,\tag{8}$$

where Σ_i represents a summation over the horizon oscillators with the temperature T_h , and the number of oscillators are proportional to the horizon area $\sim (r/k_U^{-1})^2$. At the last step we used the equipartition approximation $\hbar\omega \sim k_B T_h \sim 1/r$. Note that this is just an order of magnitude estimation for comparison, and more accurate solution requires a careful calculation with an appropriate horizon.

This result indicates that the bulk QFT overestimates the independent DOF in the bulk and the true vacuum energy of the bulk could be the zero point energy of the boundary DOF on the horizon, which is of order of the normal-ordered bulk vacuum energy in the conventional QFT. What gives the small HDE could be the smallness of the number of independent DOF in the bulk. This redundancy of the bulk DOF can explain why we cannot obtain the correct dark energy density by simply calculating the zero point energy of the bulk. In short, QFT is not a complete theory at the cosmic scale.

We need to calculate the horizon energy E_h as the vacuum energy of the universe without using QFT. Let us consider a causal cosmic horizon with a radius r, having generic holographic entropy

$$S_h = \frac{\eta c^3 r^2}{G\hbar},\tag{9}$$

and temperature

$$T_h = \frac{\epsilon \hbar c}{k_B r},\tag{10}$$

with constants η and ϵ (See Fig. 1). (Note that these quantities contain \hbar and are usually derived by semiclassical calculations.) In this case the universe is similar to a big black hole with an expanding horizon. For the Bekenstein entropy $\eta = \pi$, and the Hawking-Gibbons temperature $\epsilon = 1/2\pi$. By assuming the first law and integrating dE_h on the isothermal surface Σ of the causal horizon with Eqs. (9) and (10), we obtain the horizon energy

$$E_h = \int_{\Sigma} dE_h = k_B T_h \int_{\Sigma} dS_h = \frac{\eta \epsilon c^4 r}{G}.$$
(11)

Then, the energy density due to E_h is given by

$$\rho_h = \frac{3E_h}{4\pi r^3} = \frac{6\eta\epsilon c^3 M_P^2}{\hbar r^2} \equiv \frac{3d^2 c^3 M_P^2}{\hbar r^2},\tag{12}$$

which has the form of the holographic dark energy [16]. Note that this semiclassical derivation of HDE is different from the usual derivation based on UV-IR relations. ρ_h here corresponds to the estimation of the surface vacuum energy in Eq. (8). This kind of dark energy was also derived in terms of entanglement energy [31] and quantum entanglement force [41]. From the above equation we immediately obtain a formula for the constant

$$d = \sqrt{2\eta\epsilon},\tag{13}$$

which is the important parameter determining the nature of HDE. If S_h saturates the Bekenstein bound and T_h is the Hawking-Gibbons temperature $\hbar c/2\pi k_B r$, then $\eta \epsilon = 1/2$ and d = 1. Thus, the holographic principle applied to a cosmic causal horizon naturally leads to HDE with d = 1 [41], which is favored by observations and theories [44, 45]. There are few works on fixing d value. Li found d = 1 by assuming the universe as a *classical* black hole [16]. On the contrary in this paper d value is obtained by considering the semiclassical quantities.

Let us turn to the cosmological constant problem in this context. From the holographic viewpoint, it is very simple to see why the cosmological constant Λ should be zero. If we apply the holographic principle and the definition of the horizon energy (Eq. (11)) to the cosmic horizon, the bulk vacuum energy density ρ_{vac} in Eq. (6) should be smaller than the horizon energy density ρ_h in Eq. (12). Since the principle is one of our starting postulates, the principle should hold strictly even for arbitrary large r, and hence, the vacuum energy E_{Λ} proportional to Λr^3 is problematic. It clearly violates the holographic principle for large r, where vacuum energy E_h is proportional to r. For $r > r_c \equiv \sqrt{\frac{3d^2}{\Lambda}}$, $E_{\Lambda} > E_h$ and the holographic principle can be violated. Thus, the principle holds true for *arbitrary r* only if the cosmological constant Λ is exactly zero. Note that this argument holds for arbitrary small coefficient of r^3 term in E_{Λ} as long as r can increase infinetely. As the universe expands, the inequality $E_{\Lambda} \leq E_h$ would be violated eventually. For example, if the cosmological constant is the dark energy, constant ρ_{Λ} is about the present critical density $\rho_c = 3H^2 M_P^2 / 8\pi \propto 1/t^2$ and within a few Hubble times $\rho_{\Lambda} r^3$ will exceed $\rho_h r^3$. Thus, we can say that the holographic principle insists that the cosmological constant is zero (i.e., $\omega_{DE} \neq -1$). (Here, we have excluded an implausible case that Λ_c and a constant part of the quantum contribution miraculously cancel each other to result in $\rho_{vac} \sim 1/r^2$.)

This solution to the cosmological constant problem has its own cost. At the large cosmic scale, we have to abandon QFT and accept the holographic principle and the dark energy problem becomes much easier. As long as the principle holds, the argument about the zero cosmological constant would be valid. Since the principle also solves the other subproblems about dark energy, this approach seems to be promising.

Furthermore, the solution above is free from some difficulties often encountered by other approaches such as infrared or ultraviolet modifications of gravity, adjusting initial conditions, or dynamical attractor mechanisms (See [49] for example.). They failed to explain both of the early small universe and current large universe and why the QFT vacuum loops or cosmological phase transitions did not curl up the universe. Let us discuss these facts in detail.

First, our approach based on the holographic principle suggests that the energy density from the vacuum loop energy for a quantum field with a UV cutoff energy scale M is $\rho_M = O(M^2H^2) \ll O(M_P^2H^2)$ not of $O(M^4)$. Since the Friedmann equation is $\rho_{tot} = 3H^2M_P^2$, the vacuum loop energy is not a dominant contribution to ρ_{tot} unless $M \simeq M_P$. Second, our approach could also avoid the issue related to the cosmological phase transitions. For example, consider a phase transition of the scalar field field ϕ with a thermal effective potential $V(\phi, T)$, showing the transition at the temperature $T = T_c$. Then, in conventional approaches even if ρ_A was set to be 0 before the transition, during the phase transition the potential could generate temporally the energy difference between the false vacuum and the true vacuum, which is of $-O(T_c^4) = -O(M_P^2H^2)$, where H is the Hubble parameter at the transition. If the absolute value of energy matters, this energy difference could act as a negative cosmological constant and make the universe rapidly collapse. However, in our theory there is always positive $O(M_P^2H^2)$ dark energy that could cancel the negative energy term and prevent the collapse. Third, unlike the dynamical attractor theories, our theory does not directly rely on the contributions of matters to energy-momentum tensor and hence we do not need a feedback mechanism adjusting ρ_Λ to precision 10^{-120} as long as the horizon radius is $O(H^{-1})$.

One can easily see the similarity between our theory and entropic gravity. In entropic gravity the horizon energy is given by the equipartition law $E_h = NT_h/2$, which is essentially equivalent to our dark energy $E_h = \int T_h dS$, because $S \sim N$ in general. Following [41] and [40] one can also obtain an entropic force for the dark energy

$$F_h \equiv \frac{dE_h}{dr} = \frac{c^4 \eta \epsilon}{G},\tag{14}$$

which could be also identified as a 'quantum entanglement force' as in [41], if S_h is the entanglement entropy.

Let us compare predictions of our theory with observational data. From $\rho_{DE} = \rho_h$ and a cosmological energymomentum conservation equation, one can obtain an *effective* dark energy pressure [16] in the bulk

$$p_{DE} = \frac{d(a^3 \rho_h(r))}{-3a^2 da},$$
(15)

from which one can derive the equation of state.



FIG. 2. (Color online) Theoretical evolution of the dark energy equation of state (the blue thick line) w_{DE} versus the redshift z compared to the observational constraints (Data extracted from Fig. 2 in [50]). The green thin line represents the best fit. The dashed lines and the dotted lines shows 1σ and 2σ errors, respectively.

At this point we need to choose a horizon among various cosmological horizons such as an apparent horizon, a Hubble horizon, and a future event horizon. In the simplest case, only the event horizon can result in the accelerating universe [16]. Thus, from now on we assume the case that the causal horizon is the cosmic event horizon. For this case one can find the equation of state for holographic dark energy as a function of the redshift z as shown in Ref.

[16]. Fig. 1 compares this prediction with d = 1 to the observational data obtained from the 182 gold SN Ia data, the baryon acoustic oscillation, SDSS, and the 3-year Wilkinson Microwave Anisotropy Probe (WMAP) data. One

can also find that the equation of state [5, 16] $w_0 = -\frac{1}{3} \left(1 + \frac{2\sqrt{\Omega_{\Lambda}^0}}{d} \right)$, and its change rate at the present w_1 with

 $w_{DE}(a) \simeq w_0 + w_1(1-a)$. Here the current dark energy density parameter $\Omega_{\Lambda}^0 \simeq 0.73$.

For d = 1 these equations give $w_0 = -0.903$ and $w_1 = 0.104$. According to WMAP 5-year data [51], $w_0 = -1.04 \pm 0.13$ and $w_1 = 0.11 \pm 0.7$. WMAP 7-year data with the baryon acoustic oscillation, SN Ia, and the Hubble constant yields $w_0 = -0.93 \pm 0.13$ and $w_1 = -0.41^{+0.72}_{-0.71}$ [7].

If we use an entanglement entropy calculated in [41] for S_h , one can obtain d slightly different from 1. It is also straightforward to study the cases with other horizons such as apparent horizons or Hubble horizons. We saw that the predictions of our theory well agree with the recent observational data. Note that although the cosmological constant is most favored by the cosmological observations, the observational data still allow dynamical dark energy models.

It was also shown that holographic dark energy models with an inflation with a number of e-folds $N_e \simeq 65$ can solve the cosmic coincidence problem [16, 52] thanks to a rapid expansion of the event horizon during the inflation.

I summarize how the holographic principle and the horizon energy can solve the dark energy problem. In this theory the dark energy density is small due to the holographic principle, comparable to the critical density due to the O(1/H) horizon size, and non-zero due to the quantum fluctuation. The vacuum fluctuation energy is not huge but comparable to the observed dark energy, because conventional QFT overestimates the actual independent DOF. The holographic principle and the first law of thermodynamics also demand that the cosmological constant is zero, because the nonzero time independent cosmological constant is inconsistent with them.

Compared to previous works on HDE, our work has following new features. First, albeit simple, the dark energy theory in this paper seems to provide us a logically self-consistent explanations to the all subproblems of the dark energy including the cosmological constant problem. Second, the parameter d is obtained using semiclassical parameters such as Hawking temperature incorporating quantum effects to some extent. Third, the relations among HDE, QFT vacuum energy and entropic gravity are studied.

Note that our solution is more than a simple transformation of one problem into another one, because the formalism we used here is based not on the conventional QFT but on the holographic principle that could allow a reformulation of gravity and quantum mechanics in terms of thermodynamics as recently suggested by some authors [17, 48, 53]. Therefore, there is interesting possibility that these thermodynamic approaches could open a new route to understanding not only dark energy but also the unification of quantum mechanics and gravity in the future.

ACKNOWLEDGMENTS

This work was supported in part by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the ministry of Education, Science and Technology (2010-0024761) and the topical research program (2010-T-1) of Asia Pacific Center for Theoretical Physics.

- [3] Y. Gong, B. Wang, and Y.-Z. Zhang, Phys. Rev. D 72, 043510 (2005).
- [4] X. Zhang and F.-Q. Wu, Phys. Rev. D 72, 043524 (2005).
- [5] Q.-G. Huang and Y. Gong, JCAP **2004**, 006 (2004).
- [6] U. Seljak, A. Slosar, and P. McDonald, JCAP 0610, 014 (2006).
- [7] E. Komatsu *et al.*, arXiv:1001.4538 (2010).
- [8] P. J. E. Peebles and B. Ratra, Rev. Mod. Phys. 75, 559 (2003).
- [9] R. Bousso, Gen. Rel. Grav. 40, 607 (2008).
- [10] B. Guberina, R. Horvat, and H. Stefancic, JCAP 0505, 001 (2005).
- [11] R. Erdem, Phys. Lett. **B621**, 11 (2005).
- [12] G. 't Hooft, Salam-festschrifft (World Scientific, Singapore, 1993).
- [13] J. D. Bekenstein, Phys. Rev. D49, 1912 (1994).
- [14] A. G. Cohen, D. B. Kaplan, and A. E. Nelson, Phys. Rev. Lett. 82, 4971 (1999).
- [15] S. D. H. Hsu, Phys. Lett. B **594**, 13 (2004).
- [16] M. Li, Phys. Lett. B 603, 1 (2004).
- [17] E. P. Verlinde, arXiv:1001.0785 (2010).
- [18] T. Padmanabhan, arXiv:0912.3165 (2009).
- [19] L. Zhao, arXiv:1002.0488 (2010).

^[1] A. G. Riess *et al.*, Astron. J. **116**, 1009 (1998).

^[2] S. Perlmutter et al., Astroph. J. 517, 565 (1999).

- [20] R.-G. Cai, L.-M. Cao, and N. Ohta, arXiv:1002.1136 (2010).
- [21] Y. S. Myung, arXiv:1002.0871 (2010).
- [22] Y.-X. Liu, Y.-Q. Wang, and S.-W. Wei, arXiv:1002.1062 (2010).
- [23] Y. Tian and X. Wu, arXiv:1002.1275 (2010).
- [24] M. Diego, arXiv:1002.1941 (2010).
- [25] A. Pesci, arXiv:1002.1257 (2010).
- [26] I. V. Vancea and M. A. Santos, arXiv:1002.2454 (2010).
- [27] R. A. Konoplya, arXiv:1002.2818 (2010).
- [28] H. Culetu, arXiv:1002.3876 (2010).
- [29] Y. Zhao, arXviv:1002.4039 (2010).
- [30] S. Ghosh, arXiv:1003.0285 (2010).
- [31] J.-W. Lee, J. Lee, and H.-C. Kim, JCAP08(2007)005;hep-th/0701199 (2007).
- [32] H.-C. Kim, J.-W. Lee, and J. Lee, Mod. Phys. Lett. A 25, 1581 (2010) [arXiv:0709.3573 [hep-th]].
- [33] H.-C. Kim, J.-W. Lee, and J. Lee, JCAP 0808, 035 (2008).
- [34] J.-W. Lee, H.-C. Kim, and J. Lee, arXiv:1001.5445 (2010).
- [35] M. Li, X. D. Li, S. Wang and Y. Wang, Commun. Theor. Phys. 56, 525 (2011) [arXiv:1103.5870 [astro-ph.CO]].
- [36] J.-W. Lee, J. Lee, and H.-C. Kim, Proceedings of the National Institute for Mathematical Science 8, 1 (2007) arXiv:0709.0047 (2007).
- [37] M. Li and Y. Wang, arXiv:1001.4466 (2010).
- [38] Y. Zhang, Y.-g. Gong, and Z.-H. Zhu, arXiv:1001.4677 (2010).
- [39] S.-W. Wei, Y.-X. Liu, and Y.-Q. Wang, arXiv:1001.5238 (2010).
- [40] D. A. Easson, P. H. Frampton, and G. F. Smoot, arXiv:1002.4278 (2010).
- [41] J.-W. Lee, H.-C. Kim, and J. Lee, arXiv:1002.4568 (2010).
- [42] T. Padmanabhan, arXiv:0911.5004 (2009).
- [43] R. H. Brandenberger, Rev. Mod. Phys. 57, 1 (1985).
- [44] Q.-G. Huang and M. Li, JCAP **0408**, 013 (2004).
- [45] X. Zhang and F.-Q. Wu, arXiv:astro-ph/0701405 (2007).
- [46] J.-W. Lee, H.-C. Kim, and J. Lee, Mod. Phys. Lett. A25, 257 (2010).
- [47] E. Keski-Vakkuri and M. S. Sloth, JCAP **2003**, 001 (2003).
- [48] T. Padmanabhan, arXiv:0807.2356 (2008).
- [49] R. Bousso, Gen. Rel. Grav. 40, 607-637 (2008). [arXiv:0708.4231 [hep-th]].
- [50] Y.-G. Gong and A. Wang, Phys. Rev. **D75**, 043520 (2007).
- [51] E. Komatsu *et al.*, Astrophys. J. Suppl. **180**, 330 (2009).
- [52] J. Lee, H.-C. Kim, and J.-W. Lee, Phys. Lett. B 661, 67 (2007).
- [53] J. -W. Lee, Found. Phys. 41, 744 (2011) [arXiv:1005.2739 [hep-th]].