Dark energy from instantons

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We show that in imaginary time quantum metric fluctuations of empty space form a self-consistent de Sitter gravitational instanton that can be thought of as describing tunneling from "nothing" into de Sitter space of real time (no cosmological constant or scalar fields are needed). For the first time, this mechanism is activated to give birth to a flat inflationary Universe. For the second time, it is turned on to complete the cosmological evolution after the energy density of matter drops below the threshold (the energy density of instantons). A cosmological expansion with dark energy takes over after the scale factor exceeds this threshold, which marks the birth of dark energy at a redshift $1 + z \approx 1.3$ and provides a possible solution to the "coincidence problem". The number of gravitons which tunneled into the Universe must be of the order of 10^{122} to create the observed value of the Hubble constant. This number has nothing to do with vacuum energy, which is a possible solution to the "old cosmological constant problem". The emptying Universe should possibly complete its evolution by tunneling back to "nothing". After that, the entire scenario is repeated, and it can happen endlessly.

> "One might think this means that imaginary numbers are just a mathematical game having nothing to do with the real world. From the viewpoint of positivist philosophy, however, one cannot determine what is real. All one can do is find which mathematical models describe the universe we live in."
>
> Stephen Hawking, *The Universe in Nutshell*, Bantam Books, 2001, p. 59.

1 Introduction

The cosmological acceleration (dark energy (DE) effect) was discovered by observations of supernova SNIa by Riess et al. [1] and Perlmutter et al. [2]. Since then a number of hypotheses were advanced to explain this phenomenon (for references to original work, see, e.g., [3-7] and references therein; for a recent development see, e.g., [8]). The constraint on the equation-of-state parameter of DE $w = p_{de}/\varepsilon_{de}$ is $w = -1.08 \pm 0.1$ [9]. This equation of state corresponds to the de Sitter solution for the empty isotropic and homogeneous Universe with a nonzero positive cosmological constant Λ. Although the Λ term is consistent with the observational value of $w \approx -1$, there are well-known problems with that. The first one is the so-called "old cosmological constant problem": Why is Λ measured from observations of the order of 10^{-122} of vacuum energy density? The second one is the "coincidence problem": Why is the acceleration

happening during the contemporary epoch of matter domination? As a matter of fact, the observational constraint $w_{de} \approx -1$ tells us only that the DE equation of state is close to $p = -\varepsilon$, and this does not mean that it is necessarily due to $\Lambda \neq 0$. Such an equation of state can exist for other reasons (a well-known example is a scalar field). In the present work, we assume that the DE is of instanton origin, and such an assumption seems to be able to resolve the issues mentioned above.

In general, instantons are Euclidean solutions that mediate tunneling between two vacua (see, e.g., [10] and references therein). The idea of the present work to employ gravitational instantons of a certain type to tunnel "something" to the Lorentzian space of real time is not new. Basic works on gravitational instantons are collected in [11]. In application to quantum cosmology, the basic idea was to explain the origin of an inflationary Universe of Lorentzian signature by tunneling from a Riemannian space of Euclidean signature or from "nothing". There is the well-known work by Tryon

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[12] who was probably the first to propose that our Universe could be a vacuum fluctuation, that "... our Universe did indeed appear from nowhere" and to mention that "... such an event need not have violated any of the conventional laws of physics"; Zeldovich [13] who discussed a quantum creation of the Universe; Atkatz and Pagel [14] who proposed that "... the Universe arises as a result of quantum-mechanical barrier penetration"; the "no boundary proposal" by Hartle and Hawking [15] and a birth of the inflationary Universe by tunneling from "nothing" by Vilenkin [16] and Grishchuk and Zeldovich [17]. In the framework of quantum gravity, tunneling from a Riemannian space of Euclidean signature was considered by Gibbons and Hartle [18] and others.

In this paper, we show that this old idea can get a "second wind" due to a mechanism of tunneling based on quantum metric fluctuations, which is able to address both the birth of DE and inflationary Universe. As well as in [16], by "nothing" we mean a state with no classical space-time. Vilenkin's [16] proposal was based on the Hawking-Moss instanton [19] which gives birth to a closed inflationary universe for some models of scalar field.[2] We believe that the advantage of the mechanism proposed here lies in the fact that it is quite universal in the sense that it does not depend on the model of a scalar field but is based on natural quantum fluctuations of the space metric. It is also able to give birth to both a flat inflationary Universe to start its cosmological evolution and to DE in the contemporary epoch of aging emptying Universe to finish its cosmological evolution. This fact permits us to interpret both phenomena by a single mechanism (for similarities between primordial DE driving inflation and present DE see [20]). The Einstein equations do not fix the signature, which means that a signature change can also be considered in the framework of classical general relativity ([21–23] and others; see also [24] for a complete list of references). This approach can be thought of as a classical idealization of tunneling solutions [25].

Technically, in this paper, we find self-consistent solutions to the equations of quantum gravity in the one-loop approximation in imaginary time and then analytically continue these to the Lorentzian space of real time. Assuming that these solutions do exist in the space of real time, we show that this procedure is capable of providing a plausible interpretation for both the birth of a flat inflationary universe and the DE. In a sense, this is one of possible concrete realizations of Hawking's idea on the reality of imaginary time. We can only partly overcome the present lack of a consistent quantum theory of gravity by using the one-loop approximation in which it is finite and mathematically consistent (see [31, 32]). The one-loop approximation of quantum gravity is believed to be applicable to the modern Universe because of its remoteness from the Planck epoch. As was shown in [31, 32], the de Sitter gravitational instanton is one of three exact solutions to the exact self-consistent equations of one-loop quantum gravity that were obtained by using the BBGKY chain approach. In this paper, we obtain the same de Sitter exact solution directly from the original equations of one-loop quantum gravity (with no use of the BBGKY chain). This approach creates a window of opportunity for the new "tunneling interpretation" of this solution, which does not require "ghost materialization" in real time that was the case in [32]. Another goal of this work is to show that the tunneling of DE to the real-time Universe has favorable conditions precisely in the matter-dominated epoch (Section 4).

In Section 2, we present exact self-consistent equations of quantum gravity in the one-loop approximation in real time. We show that in real time quantum metric fluctuations are unable to form a self-consistent de Sitter solution, but they can do that if the time variable has at least an infinitesimally small imaginary part. In Section 3, we show that in imaginary time (Euclidean space) quantum metric fluctuations form a self-consistent de Sitter instanton that can be thought of as describing tunneling to the Lorentzian space of real time from "nothing". The topological non-equivalence between manifolds plays a role of a classically impenetrable barrier, quantum tunneling across which can create a flat inflationary Universe. In Section 4, we show that in the presence of matter, such tunneling is able to give birth to DE only after the density of matter drops below a critical level, and the Universe will become quite empty again. The existence of such a threshold is a possible solution to the "coincidence problem". In Section 5, we

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[2]An attempt to create a flat inflationary universe by a gravitational instanton formed by a scalar field was made in [26] but met criticism (see [27, 28]; see also [29, 30]).

outline a scenario of cosmological evolution based on the proposed mechanism.

2 Quantum metric fluctuations in real time

In this work, for curvatures much smaller than the Planck curvature, quantum cosmology is represented as a theory of gravitons in macroscopic space-time with a self-consistent geometry. The quantum state of gravitons is determined by their interaction with a macroscopic field, and the macroscopic (background) geometry, in turn, depends on the state of gravitons. The background metric and the graviton operator appearing in the self–consistent theory are extracted from the unified gravitational field, which initially satisfies the exact equations of quantum gravity. The classical component of the unified field is a function of co-ordinates and time by definition. The quantum component of the same unified field is described by a tensor operator function, which also depends on coordinates and time. Under such formulation of the problem, the original exact equations should be the operator equations of quantum theory of gravity in the Heisenberg representation. The rigorous mathematical derivation of these equations and their relation to existing references can be found in [33]. For the first time, these equations (and exact solutions) were given in [31]. Referring the reader for details to these works (see also [32], sections II, III and XII), note that an inherent part of these equations is the unavoidable appearance of the auxiliary ghost fields introduced by Feynman [34] and known as Faddeev-Popov ghosts [35]. It is precisely the appearance of ghosts that ensures the one-loop finiteness of quantum gravity, making the theory mathematically consistent ([31], [32] Sections III.D and III.E). In the self-consistent theory of gravitons, the background metric is described by regular vacuum Einstein equations [31]

$$
R_i^k - \frac{1}{2}\delta_i^k R = \kappa\big(\langle\,\Psi_g|\hat{T}_{i(\text{grav})}^k|\Psi_g\,\rangle
$$
$$
+ \langle\,\Psi_{\text{gh}}|\hat{T}_{i(\text{ghost})}^k|\Psi_{\text{gh}}\,\rangle\big). \tag{1}
$$

Here Ψ_{g}, Ψ_{gh} are quantum state vectors of gravitons and ghosts, respectively. The explicit form of the energy-momentum tensors of gravitons $\hat{T}_{i(\text{grav})}^k$ and $\hat{T}_{i(\text{ghost})}^k$ is presented in [32], Section II.F). The second term on the right-hand side of

(1) comes from Faddeev-Popov ghosts and provides one-loop finiteness of quantum gravity. We consider the self-consistent model of the Universe which is flat, isotropic and homogeneous on the average (the FLRW metric). The calculations presented here were done in the class of synchronous gauges (that automatically provide one-loop finiteness of observables). From (1) follow equations for the energy density ε_{g} and pressure p_{g} ([31] and [32], Section III.B). Heisenberg's operator equations for Fourier components of the transverse 3-tensor graviton field, Grassmann's ghost field and canonical commutation relations for gravitons and anti-commutation relations for ghosts are also presented in these papers. These equations form a self-consistent set of equations of one-loop quantum gravity for gravitons, ghosts and the FLRW background (Eqs. (2)–(4) of [31]. Another form of the same equations is presented in [32] (Eqs. (III.30-III.34)). In this paper, we use the latter, in which it is convenient to use the conformal time $\eta = \int dt/a$ and to pass on from summing to integration by the transformation

$$
\sum_k ... \to \int d^3k/(2\pi)^3... = \int_0^\infty k^2 dk/2\pi^2....
$$

These operations lead to the following set of equations:

$$
3\frac{a'^2}{a^4} = \kappa\varepsilon_g = \frac{1}{16\pi^2}\int_0^\infty \frac{k^2}{a^2}dk
$$
$$
\times\Big(\sum_\sigma\langle\,\Psi_g|\hat{\psi}_{\mathbf{k}\sigma}'^{+}\hat{\psi}_{\mathbf{k}\sigma}' + k^2\hat{\psi}_{\mathbf{k}\sigma}^+\hat{\psi}_{\mathbf{k}\sigma}|\Psi_g\,\rangle
$$
$$
- 2\langle\,\Psi_{\text{gh}}|\bar{\theta}_{\mathbf{k}}'\theta_{\mathbf{k}}' + k^2\bar{\theta}_{\mathbf{k}}\theta_{\mathbf{k}}|\Psi_{\text{gh}}\,\rangle\Big), \tag{2}
$$

$$
2\frac{a''}{a^3} - \frac{a'^2}{a^4} = -\kappa p_g = -\frac{1}{16\pi^2}\int_0^\infty \frac{k^2}{a^2}dk
$$
$$
\times\Big(\sum_\sigma\langle\,\Psi_g|\hat{\psi}_{\mathbf{k}\sigma}'^{+}\hat{\psi}_{\mathbf{k}\sigma}' - \frac{k^2}{3}\hat{\psi}_{\mathbf{k}\sigma}^+\hat{\psi}_{\mathbf{k}\sigma}|\Psi_g\,\rangle
$$
$$
- 2\langle\,\Psi_{\text{gh}}|\bar{\theta}_{\mathbf{k}}'\theta_{\mathbf{k}}' - \frac{k^2}{3}\bar{\theta}_{\mathbf{k}}\theta_{\mathbf{k}}|\Psi_{\text{gh}}\,\rangle\Big). \tag{3}
$$

And for fluctuations

$$
\hat{\phi}_{\mathbf{k},\sigma}'' + \big(k^2-a''/a\big)\hat{\phi}_{\mathbf{k},\sigma} = 0, \quad \hat{\psi}_{k\sigma} = (1/a)\hat{\phi}_{k\sigma} \tag{4}
$$

$$
\hat{\vartheta}_{\mathbf{k}}'' + \big(k^2 - a''/a\big)\hat{\vartheta}_{\mathbf{k}} = 0, \qquad \hat{\theta}_{\mathbf{k}} = (1/a)\hat{\vartheta}_{\mathbf{k}} \tag{5}
$$

Here σ is the polarization index; $a(\eta)$ is the FLRW scale factor; $\kappa = 8\pi G$; the superscript "+" denotes complex conjugation, and dots are time derivatives.

Primes denote derivatives in the conformal time η. The de Sitter expansion is

$$a_s = -(H\eta)^{-1} \tag{6}$$

In Section 3, we will need exact solutions to (4), (5) in the de Sitter background (6). They read (see [31])

$$\hat{\psi}_{\mathbf{k}\sigma} = \frac{1}{a_s}\sqrt{\frac{2\kappa\hbar}{k}}\left[\hat{c}_{\mathbf{k}\sigma}f(x) + \hat{c}^+_{-\mathbf{k}-\sigma}f^+(x)\right],$$

$$\hat{\theta}_{\mathbf{k}} = \frac{1}{a_s}\sqrt{\frac{2\kappa\hbar}{k}}\left[\hat{\alpha}_{\mathbf{k}}f(x) + \hat{\bar{\beta}}_{-\mathbf{k}}f^+(x)\right], \tag{7}$$

$$f(x) = (1 - i/x)e^{-ix}, \qquad x = k\eta. \tag{8}$$

Quantum metric fluctuations (7), (8) are unable to form a self-consistent de Sitter solution to Eqs. (7)–(8) in real time because of incomputability of the integrals $\int_0^\infty x^n e^{\pm 2ix}dx$ arising in the right-hand side of (2), (3). However, they can do that if these incomputable integrals are redefined as [31]

$$\int_0^\infty x^n e^{\pm 2ix}dx = \lim_{\delta\to 0}\int_0^\infty x^n e^{\pm 2ix - \delta x}dx.$$

Such a re-definition implies that the variable $x = k\eta$ must have at least an infinitesimally small imaginary part to be able to form a self-consistent de Sitter solution. In the next section, we show that the self-consistent de Sitter solution does exist in imaginary time, i.e., in Riemannian space of Euclidean signature from which it can be analytically continued to the Lorentzian space of real time.

3 Quantum metric fluctuations in imaginary time

The transition to imaginary time in Eqs. (2)-(5) is carried out by replacing the variables:

$$t = -i\tau, \qquad \eta = -i\upsilon \tag{9}$$

The transition (9) transforms (2-5) to the following set of equations:

$$-3\frac{a'^2}{a^4} = \frac{1}{16\pi^2}\int_0^\infty \frac{k^2}{a^2}dk$$
$$\times\left(\sum_\sigma\langle\Psi_g| - \hat{\psi}'^+_{k\sigma}\hat{\psi}'_{k\sigma} + k^2\hat{\psi}^+_{k\sigma}\hat{\psi}_{k\sigma}|\Psi_g\rangle\right.$$
$$\left. - 2\langle\Psi_{\mathrm{gh}}| - \bar{\theta}'_k\theta'_k + k^2\bar{\theta}_k\theta_k|\Psi_{\mathrm{gh}}\rangle\right) \tag{10}$$

$$\hat{\phi}''_{\mathbf{k},\sigma} - (k^2 + a''/a)\hat{\phi}_{\mathbf{k},\sigma} = 0, \quad \hat{\psi}_{k\sigma} = \frac{1}{a}\hat{\phi}_{k\sigma}; \tag{11}$$

$$\hat{\vartheta}''_{\mathbf{k}} - (k^2 + a''/a)\hat{\vartheta}_{\mathbf{k}} = 0, \qquad \hat{\theta}_k = (1/a)\hat{\vartheta}_{\bar{k}}. \tag{12}$$

Primes in this section denote derivatives in the imaginary conformal time υ. After such a transition, solutions to (10)–(12) over de Sitter background read

$$\hat{\psi}_{k\sigma} = \frac{1}{a}\sqrt{\frac{2\kappa\hbar}{k}}(\hat{Q}_{\kappa\sigma}g_k + \hat{p}_{k\sigma}h_k),$$

$$\hat{\theta}_k = \frac{1}{a}\sqrt{\frac{2\kappa\hbar}{k}}(\hat{q}_\kappa g_k + \hat{p}_k h_k), \tag{13}$$

where

$$g(\xi) = (1 + 1/\xi)e^{-\xi}, \qquad h(\xi) = (1 - 1/\xi)e^\xi,$$
$$\xi = k\upsilon. \tag{14}$$

The requirement of finiteness eliminates the $h-$ solution. The operator functions (13) can be named quantum fields of gravitational instantons of graviton and ghost type, or for short, graviton-ghost instantons. Substitution of (13)-(14) into the right-hand side of (10) leads to

$$3\frac{a'^2}{a^4} = \frac{\kappa\hbar H_\tau^4}{2\pi^2}\int_0^\infty N_k[\xi^2 - (1+\xi)^2]e^{-2\xi}\xi d\xi, \tag{15}$$

$$N_k = \sum_\sigma\langle\Psi_g|\hat{Q}^+_{k\sigma}Q_{k\sigma}|\Psi_g\rangle$$
$$- 2\langle\Psi_{\mathrm{gh}}|\hat{q}^+_k\hat{q}_k|\Psi_{\mathrm{gh}}\rangle. \tag{16}$$

Here H_τ is the Hubble function in imaginary time. The de Sitter background in imaginary time is

$$a_s = -(H_\tau\upsilon)^{-1} \tag{17}$$

In the de Sitter background (17), the left-hand side of (15) must be $3H_\tau^2 = \mathrm{const}$, which means that the right-hand side of (15) cannot be a function of υ. In turn, this means that only a flat spectrum $N_k = \mathrm{const}$ is able to provide constancy of the right-hand side of (15). To make the result more transparent, assume that the spectrum is flat and the numbers of instantons of ghost and anti-ghost type are equal to each other, i.e., $\langle n_{k(g)}\rangle = \langle n_g\rangle$; $\langle n_{k(\mathrm{gh})}\rangle = \langle \bar{n}_{\mathrm{gh}}\rangle = \langle n_{\mathrm{gh}}\rangle$. Assume also that typical occupation numbers in the ensemble are large, so that squares of modules of probability amplitudes are likely to be described by Poisson distributions. In such a case, we get a simple physically transparent result ([32], section VII):

$$N_k = 4(\langle n_g\rangle - \langle n_{\mathrm{gh}}\rangle). \tag{18}$$

The de Sitter solution (17) satisfies Eq. (15) if H_τ satisfies Eq. (19):

$$H_\tau^2 \left(1 + \frac{\kappa \hbar H_\tau^2 (\langle n_g \rangle - \langle n_{\text{gh}} \rangle)}{8\pi^2} \right) = 0. \qquad (19)$$

A real solution to (19) exists if $\langle n_{(g)} \rangle - \langle n_{\text{gh}} \rangle \leq 0$. It reads

$$H_\tau^2 = \frac{8\pi^2}{\kappa \hbar (\langle n_{\text{gh}} \rangle - \langle n_g \rangle)} \quad \text{if} \quad H_\tau^2 \neq 0,$$

$$H_\tau^2 = 0 \quad \text{if} \quad H_\tau^2 \neq \frac{8\pi^2}{\kappa \hbar (\langle n_{\text{gh}} \rangle - \langle n_g \rangle)}. \qquad (20)$$

In real time, ghosts are fictitious particles which appear to compensate the spurious effect of vacuum polarization of fictitious fields of inertia. In real time, the gravitational effect of gravitons is proportional to $(\langle n_g \rangle - \langle n_{\text{gh}} \rangle) \geq 0$ and, figuratively speaking, this means that those ghosts cannot be "materialized" in real time. The solution (20) tells us that in imaginary time ghosts are "materialized" to form a self-consistent de Sitter solution. A remarkable fact is that a passage to the Lorentzian space of real time "de-materializes" the ghosts (see below), so that since the analytic continuation is done, the difference (18) is positive again in the Lorentzian space, and ghosts are again fictitious particles as they must be.[3] The next step is to analytically continue the imaginary-time solution (17) to the space of real time. To do so, we analytically continue (17) from the imaginary axis υ to the plane of complex conformal time $\varsigma = i\eta + \upsilon$. It reads

$$a(\varsigma) = -(H\varsigma)^{-1}. \qquad (21)$$

Here

$$H = \pm \left(\frac{8\pi^2}{\kappa \hbar \langle n_g \rangle - \langle n_{\text{gh}} \rangle} \right)^{1/2} = \pm i H_\tau. \qquad (22)$$

Assuming that the number of gravitons in the Universe is $N \approx \langle n_g \rangle - \langle n_{\text{gh}} \rangle$, we can rewrite H as

$$H = \pm \left(\frac{8\pi^2}{\kappa \hbar N} \right)^{1/2}. \qquad (23)$$

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[3]Ghost "materialization" in imaginary time does not affect physical quantities in real time. A good example is the creation of electron-positron pairs in a constant electric field [16]. Energy conservation for the electron is given by the equation $m(1 - \dot{x}^2)^{1/2} - eEx = \text{const}$. To compute the probability of pair creation, one needs a transition to imaginary time which leads to $m(1 + \dot{x}^2)^{1/2} - eEx = \text{const}$. The formal appearance of superluminal motion in imaginary time does not affect the physical quantities in real time.

It follows from (21), (22) that on the imaginary axis $\eta = 0, \upsilon \neq 0$ one gets (17), and on the real axis $\eta \neq 0, \upsilon = 0$, one gets (6). Thus we arrive at the identity

$$H_\tau^2 \upsilon^2 \equiv H^2 \eta^2 \qquad (24)$$

At the origin $\varsigma = 0 + i0$, we choose the second solution of (20), $H = H_\tau = 0$, which provides the junction condition $(a'/a^2)_{\varsigma=0} = 0$ at the boundary of signature change (see [22, 23]). In the same figurative words, the analytic continuation to real time "de-materializes" the ghosts, and in real Lorentzian space they become fictitious particles.[4] The numerical value of N is of the order of the number of gravitons tunneled into de Sitter space of real time by instantons (see below). In the case of the Poisson distributions (used above), $N^{-1} \sim \langle (\Delta N/N)^2 \rangle$, where ΔN is a fluctuation of the number of gravitons. This means that $H^2 \sim \langle (\Delta N/N)^2 \rangle$, so that the speed of Hubble expansion is governed by quantum metric fluctuations, as expected. From (15) follows the equation of state

$$\varepsilon_g = -p_g = \frac{3\hbar N}{8\pi^2} H^4.$$

This equation of state is superficially similar to what comes from quantum conformal anomalies. As was shown by Starobinsky [36], quantum corrections to the Einstein equations due to zero oscillations can provide a self-consistent de Sitter solution in the vicinity of Planck's value of curvature (see also [37]). In such a case, the equation of state is $\varepsilon \sim n\hbar H^4$ [38], where the number of types of elementary particles n is ≤ 100. Conformal anomalies that arise due to regularization and renormalization procedures do not apply to this work, which deals with the equations of quantum gravity that are finite in the one-loop approximation. In the finite one-loop quantum gravity, the effect of conformal anomalies is exactly zero, and the de Sitter solution can be formed only by graviton-ghost instantons ([32], Sec. XII). In contrast to the conformal anomaly parameters, the parameter N is arbitrary and can be a huge number.

Using a mathematical analogy between (4), (5), (11), (12) and the stationary Schrödinger equation, solutions to these can be thought of in terms of

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[4]This fact distinguishes the interpretation of the de Sitter exact solution in this paper from [32], where "ghost materialization" takes place in real time.

quantum tunneling. In these equations, $x = k\eta$ plays the role of the spatial coordinate of the Schrödinger equation, and the role of a "one-dimensional potential" is played by a''/a. Whether (4)–(5) belong to Lorentzian and (11)–(12) to Euclidean space is governed by the sign of k^2 (where $+k^2$ is for real and $-k^2$ is for imaginary time). Superhorizon gravitons and ghosts ($|x^2| \ll 1$) do not "feel" the difference between Lorentzian and Euclidean signatures and can belong to each of these. This means that the boundary $x = 0$ plays the role of a classically impenetrable barrier dividing these topologically non-equivalent spaces. To complete the analytic continuation of the self-consistent solution (13) and (17) from imaginary to real time, we should do that for the graviton and ghost mode functions. To do so, we analytically continue $g(\xi)$ of (14) from the imaginary axis v to the plane of complex conformal time $\varsigma = i\eta + v$. It reads

$$g(\zeta) = \left(1 + \frac{1}{\zeta}\right) e^{-\zeta}, \quad \zeta = k\varsigma = ix + \xi. \qquad (25)$$

It follows from (25) that on the imaginary axis $x = 0$, $\xi \neq 0$ $g(\zeta) = g(\xi)$ in accordance with (14), while on the real axis $x \neq 0$, $\xi = 0$, one gets $g(\zeta) = f(x)$ from (8). The latter is an analytical continuation of $g(\xi)$ from (14) to the space of real time. Thus, in the space of real time one gets a self-consistent solution consisting of (6) and (7). The junction conditions on the boundary of signature change are $\dot{\hat{\psi}}_{\mathbf{k}\sigma} = \dot{\hat{\vartheta}}_{\mathbf{k}} = 0$ [23]. At the origin $\zeta = i0 + 0$, they are satisfied automatically for the mode functions g/a and f/a.

To decide whether the de Sitter gravitational instanton is physically allowed, one needs to calculate the Euclidean action S_E to make sure that it is finite. The action S_E, as defined in 4D space with a positive signature, reads ([32], Sec. VII.A.2)

$$S_E = \frac{1}{\kappa} \int d\tau \left\{ 3 \left[a^2 \frac{d^2a}{d\tau^2} + a \left(\frac{da}{d\tau} \right)^2 \right] \right.$$
$$+ \frac{1}{8} \sum_{\mathbf{k}\sigma} \left(a^3 \frac{d\hat{\psi}^+_{\mathbf{k}\sigma}}{d\tau} \frac{d\hat{\psi}_{\mathbf{k}\sigma}}{d\tau} + ak^2 \hat{\psi}^+_{\mathbf{k}\sigma} \hat{\psi}_{\mathbf{k}\sigma} \right)$$
$$\left. - \frac{1}{4} \sum_{\mathbf{k}} \left(a^3 \frac{d\hat{\theta}^+_{\mathbf{k}}}{d\tau} \frac{d\hat{\theta}_{\mathbf{k}}}{d\tau} + ak^2 \hat{\theta}^+_{\mathbf{k}} \hat{\theta}_{\mathbf{k}} \right) \right\}. \qquad (26)$$

The substitution of $a, \hat{\psi}_{\mathbf{k}\sigma}$ and $\hat{\vartheta}_{\mathbf{k}}$ from (13) and (14) to (26) transforms the integrand to the left-hand side of Eq. (19), which is identically zero.

The finiteness of Euclidean action justifies the fact that the self-consistent solution (13), (14) and (17) can be thought of as a gravitational instanton. The tunneling probability is proportional to $\exp(-S_E)$, which is unity in this case and means that the macroscopic evolution of the Universe is determined. The existence of the analytic continuation and finiteness of S_E allow for proposing the birth of a flat inflationary Universe by tunneling from "nothing" by means of a de Sitter gravitational instanton built up by quantum metric fluctuations.[5] Note that the self-consistent solution obtained is valid in the applicability of the one-loop approximation because it is formed by over-horizon metric fluctuations. Therefore, it is likely to be regarded as a qualitative result showing a possible path of the cosmological evolution of the Universe. The analysis of further evolution of a new inflating Universe born in such a way is the subject of inflation theory (see, e.g., [39] for references to original works and [40] for an analysis of current problems). It is useful to recall here that "so far, the details of inflation are unknown, and the whole idea of inflation remains a speculation, though one that is increasingly plausible" ([41], p. 202). Recall that the Hubble constant H is determined by the number of gravitons N that were tunneled into de Sitter space by instantons carrying out the tunneling. In the case of DE birth (Section 4), for the observed value of the Hubble constant $H = 73.8 \pm 2.4$ km$\cdot$s$^{-1}\cdot$Mpc^{-1} [9], from (24) one gets $N \sim 10^{122}$ (a numerical coefficient of the order of unity is omitted). If the cosmological constant Λ is due to the vacuum energy, as generally accepted, then $L_p^2/\Lambda^{-1} \approx 10^{-122}$, where $L_p = \sqrt{G\hbar/c^3}$ is the Planck length, and it is in direct contradiction to observations. Although there are several attempts to avoid this problem (see, e.g., [42]), it still exist. As follows from (23), an alternative interpretation of this huge number is that it is simply the number of gravitons that have tunneled into the contemporary Universe, and it has nothing to do with the vacuum energy.

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[5]The present lack of a consistent quantum theory of gravity leads to the fact that the appearance of self-consistent de Sitter space in real time by tunneling admits ambiguous physical interpretations. According to [32], Sec. VII.2 and VII.3.B, the same solution (17) can be obtained by a procedure borrowed from Quantum Chromodynamics which corresponds to a different type of tunneling.

4 Birth of dark energy

It follows from the observational data that the Universe consists of approximately 70% of DE and 30% of dark matter plus ordinary matter in the present epoch. In the previous section, we showed that tunneling from "nothing" may create the de Sitter expansion of the empty Universe in real time. In this section, we consider this process in the presence of matter. The aim of this section is to address the "coincidence problem" and the specific features of the matter-dominated epoch that distinguishes it from others.

In such a case, Eq. (2) reads

$$3\frac{a'^2\eta}{a^4} = \kappa(\varepsilon_g + \varepsilon_m), \quad \kappa\varepsilon_m = \frac{3C_m}{a^3},$$
$$3C_m = \kappa\varepsilon_m a^3 = \kappa\varepsilon_{0m}a_0^3 = \text{const.} \quad (27)$$

Here ε_{0m} and a_0 are the present energy density of matter and scale factor, respectively, and ε_g is taken from (2). In accordance with the general idea of this work, one must obtain a solution to the set of equations (27), (4) and (15) in imaginary time and then analytically continue it to the Lorentzian space of real time. In imaginary conformal time ς, (27) reads

$$3\frac{a'^2}{a^4} = -\kappa(\varepsilon_{\text{inst}} + \varepsilon_m). \quad (28)$$

The energy density of graviton-ghost instantons $\kappa\varepsilon_{\text{inst}}$ is here the right-hand side of Eq. (10). Primes in this section denote derivatives in the imaginary conformal time υ. Solutions to Eq. (28) can exist only if $\varepsilon_{\text{inst}} < 0$, i.e., again under the condition that ghosts are "materialized" in Euclidean space. This condition is necessary but not sufficient. In the presence of matter, solutions to (28) can exist only after the scale factor $a(\varsigma)$ exceeds the threshold $a \geq a_{\text{treshold}} \equiv a_T$, which is determined by the condition $\varepsilon_m \leq -\varepsilon_{\text{inst}}$. In the framework of our assumption that the DE is of instanton origin and that the imaginary-time solution can be analytically continued to real time, this threshold condition is transformed to $\varepsilon_m \leq \varepsilon_{\text{de}}$. As was mentioned in Section 1, the observational data are consistent with the de Sitter expansion law, so that $\varepsilon_{\text{de}} \approx \text{const.}$[6] From this fact and (27) it

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[6]In the general case, if $\varepsilon_{\text{de}} = \varepsilon_{\text{de}}(t) \neq \text{const}$, the threshold condition can be redefined as $\varepsilon_m \leq \varepsilon_{\text{de}}^{(\min)}$, where $\varepsilon_{\text{de}}^{(\min)}$ is the deepest minimum of ε_{de} (see the figure).

follows that for $z_{\text{theshold}} \equiv z_T$ one gets

$$1 + z_T \leq \left(\frac{\varepsilon_{\text{de}}}{\varepsilon_m}\right)^{1/3} = \left(\frac{\Omega_{\text{de}}}{\Omega_m}\right)^{1/3}. \quad (29)$$

The last term in (29) is presented with the generally accepted notation where Ω_{de} and Ω_m are the ratios of DE density and matter density to the total density of the Universe, respectively, so that $\Omega_{\text{de}} + \Omega_m = 1$. Assuming that $\Omega_{\text{de}} \approx 0.69$ [43], one gets $1 + z_T \leq 1.3$. In the existence of such a threshold lies a possible answer to the question: why the birth of DE occurred "recently", i.e. after its energy density became comparable with the energy density of matter ε_m?

The decelerated expansion changed to an accelerated one in real time at the transition point z_t where $\ddot{a} = 0$. According to [44], $z_t = 0.35 \pm 0.07$; by [45] $z_t \approx 0.29^{+0.07}_{-0.06}$ to $z_t \approx 0.60^{+0.06}_{-0.08}$; by [46], $z_t \sim 0.3$; by [47], $z_t \approx 0.43 \pm 0.07$; by [32], Sec. IX, $z_t \approx 0.34 \pm 0.02$; by [48], $z_t \approx 0.78^{+0.08}_{-0.27}$. Blake et al. [49] and Busca et al. [50] did not estimate z_t but found that an acceleration can take place for $z < 0.7$. In the case of a Λ term and the same assumption $\Omega_{\text{de}} \approx 0.69$, the condition $\ddot{a} = 0$ leads to the transition point redshift

$$z_t^{(\Lambda)} = \left(\frac{2\varepsilon_\Lambda}{\varepsilon_m}\right)^{1/3} - 1 = \left(\frac{2\Omega_\Lambda}{\Omega_m}\right)^{1/3} - 1 = 0.64. \quad (30)$$

The presence of large systematic and statistical errors makes it difficult to distinguish between z_t that come from the observational data listed above and $z_t^{(\Lambda)}$ that comes from a Λ-term theory. We would like to emphasize once again that although both the cosmological constant and instanton DE asymptotically approach the same de Sitter regime where the impact of matter becomes negligible, but this does not mean that in the transition region, where the DE and matter densities are comparable, transition points must be the same. Most likely they should differ because Eqs. (27) are different for DE of instanton origin and a cosmological constant. To see that, one can combine Eqs. (2) and (3) and pass over from the conformal real time to the physical imaginary time τ. As a result, one gets

$$\frac{\ddot{a}_\tau}{a} = \frac{\kappa}{6}(\varepsilon_m + \varepsilon_{\text{inst}} + 3p_{\text{inst}}). \quad (31)$$

If $p_{\text{inst}} \to 0$ in the vicinity of the threshold point, then the difference between z_t and z_T also tends

to zero, and this is often the case, as shown by numerical experiments (see the figure). Unlike the cosmological constant case, where z_t is about twice as large as z_T, they can be close to each other in the case of DE of instanton origin.

Unlike Section 3, where we had an exact and explicit de Sitter solution, we have no explicit solutions to the set of equations (11), (12) and (28) here to analytically continue them to the space of real time. To get numerical solutions in real time, we can employ the BBGKY-chain built on Eqs. (27), (4), (5). In an explicit form, they are Eqs. (VIII.2)–(VIII.4) of [32]. The output is a numerical solution in real time. The figure shows a typical real-time numerical solution by means of a BBGK-chain for rather arbitrary initial conditions.[7] At the beginning, oscillations can be seen, whose nature depends on the initial conditions. Then, oscillations decay, and the solution asymptotically approaches a de Sitter expansion mode when the influence of matter becomes negligible. The exit to the de Sitter regime with decreasing matter input, regardless of the specific initial conditions (if the latter correspond to the de Sitter attractor) is not a specific property of the present numerical example. It is a general property of the theory of DE of instanton origin. To show that, let us consider an approximate but explicit solution to Eq. (28). It can be obtained in the asymptotic case $|\varepsilon| \gg \varepsilon_m$ when the energy density of matter is small as compared to that of instantons. In such a case, one can replace $-\kappa\varepsilon_{\text{inst}} \approx 3H_\tau^2 = \text{const}$ in Eq. (28), where H_τ^2 is defined by (20).

Such a replacement, meaning closeness of the initial conditions to the de Sitter attractor, leads to Eq. (32) instead of (28), which can be solved explicitly:

$$3\frac{\dot{a}_\tau^2}{a^2} = 3H_\tau^2 - \frac{3C_m}{a^3} \qquad (32)$$

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[7]As was shown in [32], section V, the BBGK chain built on Eqs. (2), (4) and (5) has at least three attractors, and the de Sitter instanton is just one of them. Accordingly, the asymptotic output on this or another attractor is determined by the choice of initial conditions. Therefore, in the case shown in the figure, the initial conditions are rather arbitrary but within those leading to the de Sitter attractor.

The solution to (32) reads

$$a(\tau) = a_0\left[\cosh\left(\frac{3}{2}H_\tau\tau\right) + \sqrt{1 - \frac{C_m}{a_0^3 H_\tau^2}}\sinh\left(\frac{3}{2}H_\tau\tau\right)\right]^{2/3}. \qquad (33)$$

In the absence of matter $(C_m = 0)$, the solution (33) transforms into (17), i.e., pure de Sitter expansion. In the presence of matter $(C_m \neq 0)$, the solution (33) clearly demonstrates the existence of the threshold $C_m/(a_0^3 H_\tau^2) = \varepsilon_{0,m}/|\varepsilon_{\text{inst}}| \leq 1$. A transition to real time in (33), taking into account (9) and (22), leads to

$$a(t) = a_0\left[\cosh\left(\frac{3}{2}Ht\right) + \sqrt{1 + \frac{C_m}{a_0^3 H^2}}\sinh\left(\frac{3}{2}Ht\right)\right]^{2/3}. \qquad (34)$$

Indeed, it coincides with the well-known solution for a cosmological model containing a positive cosmological constant in the presence of non-relativistic matter. In the latter case, this solution is valid for any instant of time. Unlike that, in our case, the Eq. (32) itself, its solution (33) and the real time solution (34) are valid only asymptotically when the contribution of matter is small. Asymptotically, for $3H_\tau\tau/2 \gg 1$ and $3Ht/2 \gg 1$, one gets from (33), (34)

$$a(\tau) = a_0 e^{H_\tau\tau}\left[\frac{1}{2}\left(1 + \sqrt{1 - \frac{C_m}{a_0^3 H_\tau^2}}\right)\right]^{2/3}, \qquad (35)$$

$$a(t) = a_0 e^{Ht}\left[\frac{1}{2}\left(1 + \sqrt{1 + \frac{C_m}{a_0^3 H^2}}\right)\right]^{2/3}. \qquad (36)$$

In conformal time, (35) and (36) become (17) and (6), respectively, and the identity (24) provides again a possibility of analytical continuation of (17) to (6) over the horizon $\eta = 0$. Thus, with increasing of the scale factor, its behavior is getting asymptotically closer and closer to the de Sitter regime. The smaller the contribution of matter to the energy balance, the more conditions for tunneling approach those for empty space, and the latter are satisfied exactly in accordance with section 3. Apart from having a threshold and the de Sitter asymptotic behavior, there is another specific feature of the matter-dominated era, which distinguishes it from other eras. It is that DE tunneling
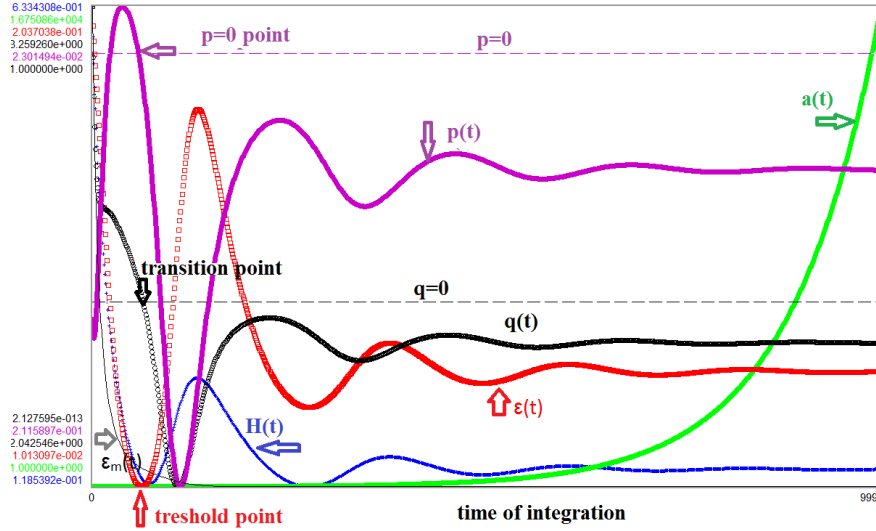
Figure 1: A real-time numerical solution is shown for the cosmological substrate consisting of DE of instanton origin and cold dark matter. Here $a(t)$ is the scale factor; $H(t) = \dot{a}/a$ is the Hubble function; $q(t) = -\ddot{a}/a$ is the acceleration/deceleration parameter; $\varepsilon_m(t)$ is the energy density of matter; $\varepsilon(t)$ and $p(t)$ are the energy density and pressure of DE. All functions are dimensionless but plotted in different scales for the sake of clarity. Numbers along the Y axis indicate numerical values of maximal (upper group) and minimal (lower group) values of the appropriate functions. The upper and lower dashed lines correspond to the zeros of the functions $p(t)$ and $q(t)$, respectively. At the transition point, the scale factor is $a_t = 2.99$. Here $q(t)$ passes through zero, changes its sign, and it is shifting away from deceleration ($q(t) > 0$) to acceleration ($q(t) < 0$). In the vicinity of this point, the pressure p passes through zero at the scale factor $a_p = 2.84$, and ε approaches its deepest minimum at the threshold point (the scale factor $a_T = 2.96$). For this particular example, one gets $(a_T - a_t)/a_t \approx 0.01$ and $(a_{p=0} - a_t)/a_t \approx 0.05$. So, $z_t - z_T = (\Delta a/a)(1 + z_t) \approx 0.01(1 + z_t)$, and $z_{p=0} - z_t \approx 0.05(1 + z_t)$, respectively. All these differences lie within the range of observational errors. As can be seen from this graph, the solution oscillates from the start. The nature of these oscillations (the number of maxima and minima, their amplitudes, etc.) depends on the choice of initial conditions which are unknown. Asymptotically, however, all such solutions come to the de Sitter mode $H \to const$; $-q \to H^2$; $\varepsilon \to 3H^2$; $p \to -3H^2$. This fact is independent of specific initial conditions if they are of the type that corresponds to the de Sitter attractor.

to the real-time Universe has favorable conditions precisely in this era (see below).

To complete the analytic continuation of the self-consistent solution, we must carry the graviton and ghost mode functions across the barrier $x = 0$ in the presence of matter. As was already mentioned, using a mathematical analogy between (4), (5), (11), (12) and the stationary Schrödinger equation, solutions to these can be thought of in terms of quantum tunneling. For $a = const \cdot \eta^{-\beta}$, the "one-dimensional potential" is $a''/a = \beta(\beta + 1)/\eta^2$ [51]. A remarkable fact is that the "one-dimensional potentials" $a''/a = 2/\eta^2$ are the same in both cases $\beta = -2$ (a matter dominated background with the equation of state $p = 0$) and $\beta = 1$ (a de Sitter background with the equation of state $p = -\varepsilon$). The same is also true for the imaginary time ς. Since the "one-dimensional potentials" coincide,

the Schrödinger-like equations (4), (5) for gravitons and ghosts over the matter dominated background and over the de Sitter background are identical. The same is true for Eqs. (11), (12). Due to identity of equations for matter-dominated and de Sitter backgrounds, the boundary conditions for tunneling are naturally satisfied at the barrier. It can be seen from two opposite limiting cases $\varepsilon_m \ll |\varepsilon_{\text{inst}}|$ and $\varepsilon_m \gg |\varepsilon_{\text{inst}}|$.

As was shown in Section 3, in the first limiting case, the boundary conditions are satisfied by the functions $f(x)$ from (7), (8), and $g(\xi)$ from (14), which correspond to the solutions of equations (4), (5), (11) and (12) in the de Sitter background, $a \sim \eta^{-1}$ in real time and $a \sim \upsilon^{-1}$ in imaginary time. In the second limiting case, one gets the same function $f(x)$ in the matter-dominated background $a \sim \eta^2$ and $g(\xi)$ in the de Sitter background $a \sim \upsilon^{-1}$.

In the case of interest (the first limiting case), we can consider (17) and (25) as approximations valid for scale factors that are close to de Sitter ones in both imaginary and real times. The combination of the two facts, the existence of a threshold and coincidence of "one-dimensional potentials", distinguishes the matter-dominated epoch from others. As a result, we arrive at the following picture. With a decreasing contribution of matter, the Universe is increasingly emptied, and conditions for tunneling approach those for empty space. Because of the identity (24), nothing prevents an empty Universe from tunneling back to "nothing" at the end of its cosmological evolution.

5 Conclusion

In imaginary time, quantum metric fluctuations of empty Euclidean space form an exact solution to the self-consistent equations of quantum gravity in the one-loop approximation that can be thought of as a de Sitter gravitational instanton. This solution is analytically continued into the Lorentzian space of real time where it gives rise to a de Sitter expansion. In the presence of matter, the same effect is switched on after the energy density of matter drops below a threshold. The following scenario can be proposed. A flat inflationary Universe could have been formed by tunneling from "nothing". After that it should evolve according to inflation scenarios that are beyond the scope of this paper. Then the standard Big Bang cosmology starts and lasts as long as the Universe begins to become empty again. As the Universe ages and is emptied, the same mechanism of tunneling that gave rise to the empty Universe at the beginning, gives now birth to dark energy. This mechanism is switched on after the energy density of matter has dropped below a critical level. After that, to the extent that the space continues to be empty, the expansion proceeds faster and faster and gradually becomes again exponentially fast (de Sitter). The identity (24) provides a possibility for the empty Universe (that has completed its cosmological evolution) to be able to tunnel back to "nothing". After that, the entire scenario can be repeated indefinitely.

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