Scenario of Accelerating Universe from the Phenomenological Λ Models

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Abstract Dark matter, the major component of the matter content of the Universe, played a significant role at early stages during structure formation. But at present the Universe is dark energy dominated as well as accelerating. Here, the presence of dark energy has been established by including a time-dependent Λ term in the Einstein's field equations. This model is compatible with the idea of an accelerating Universe so far as the value of the deceleration parameter is concerned. Possibility of a change in sign of the deceleration parameter is also discussed. The impact of considering the speed of light as variable in the field equations has also been investigated by using a well known time-dependent Λ model.

Keywords general relativity; dark matter; dark energy; accelerating Universe, VSL

1 Introduction

Dark matter and dark energy are two major constituents of the present Universe. As it stands today,

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the Universe is composed of nearly 30% matter and 70% dark energy (Kirshner 2003). Again, of the total matter content, about 25% are non-luminous or dark while baryonic matter contributes with the 5% of all energetic content of the Universe and surprisingly its largest fraction is found in the Intergalactic Medium, not in the galaxies (Ostriker and Steinhardt 2003; Shull 2005; Ettori et al. 2009). Long ago, using Virial Theorem, Zwicky (1937) for the first time suggested about the existence of dark matter. Afterwards, galactic rotation curve studies also supported Zwicky's idea (Roberts and Rots 1973; Ostriker et al. 1974; Einasto et al. 1974; Rubin et al. 1978). When, as a consequence of Inflationary Theory of Guth (1981) and others (Linde 1982; Albrecht and Steinhardt 1982), it became clear that the Universe must be flat, cosmologists became convinced that 96% matter content of the Universe should be hidden mass. But, this simplified cosmological picture soon ran into trouble since, in spite of an intense search, evidence in favor of such a huge amount of dark matter was lacking. Theoretical scientists then speculated that matter energy density of the Universe cannot be more than one-third of the total energy density and hence the remaining two-third energy density should be compensated by a cosmological constant (Turner 2003). Finally, observational result for an accelerating Universe (Perlmutter et al. 1998; Riess et al. 1998) favored the above speculation and the idea of an accelerating agent, termed as dark energy, was accepted.

Both dark matter and dark energy have played their specific roles in different stages of cosmic evolution. Dark matter had a significant role in the early Universe during structure formation because its nature is to clump in sub-megaparsec scales. Although the exact composition of dark matter is still unknown, COBE and CMB experiments suggest that baryonic dark matter cannot be more than a small fraction of total dark

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matter present in the Universe (Sahni 2004). Moreover, observational constraint regarding neutrino mass and relic neutrino density (Minakata and Sugiyama 2002; Elgaroy et al. 2002; Spergel et al. 2003; Ellis 2003) eliminate the possibility of hot dark matter. So, cold non-baryonic dark matter which can clump on small scales is favoured now (Sahni 2004). The Standard Cold Dark Matter (SCDM) model, introduced in the early 1980's, is presently disfavoured and it is replaced by Λ -CDM model in the context of an accelerating Universe. Λ -CDM model is found to be in nice agreement with various observational results (Tegmark et al. 2003) and as an advantage of assuming a nearly scale-invariant primordial perturbations and a Universe with no spatial curvature as predicted by the Inflationary theory (Mukhanov and Chibisov 1981; Guth 1982; Hawking 1982; Starobinsky 1982; Bardeen et al. 1982). But, in the Λ -CDM scenario the present acceleration of the Universe cannot be a permanent feature because, structure formation cannot proceed during acceleration. In fact, some recent works (Padmanabhan and Raychowdhury 2002; Amendola 2003) show that the present accelerating phase was preceded by a decelerating one and observational evidence (Riess 2001) also supports this idea. So, the deceleration parameter must have undergone a flip in sign during cosmic evolution.

Let us now move towards the cosmological constant problem. There are actually two fine-tuning problems with cosmological constant: (i) the value of Λ must be 123 orders of magnitude and 55 orders of magnitude larger on the Planck scale ($T \sim 10^{19}$ GeV) and the electroweak scale ($T \sim 10^2$ GeV), respectively, than its presently observed value, and (ii) the matter and radiation energy densities of the expanding Universe fall off as a^{-3} and a^{-4} , respectively, where a is the scale factor of the universe, while Λ remains constant. The only solutions to these problems is to assume a dynamical character of constant Λ , especially a time-dependent Λ which has decreased slowly from its large initial value to reach its present small value (Overduin and Cooperstock 1998). This idea of timevarying cosmological constant Λ gives us a motivation behind the present investigation.

Various dark energy models have been proposed during the last decade or so (for an overview see the works of Overduin and Cooperstock (1998) and Sahni and Starobinsky (2000)). One of the favorite candidates among these, obviously, are the models related to dynamic Λ term. In a recent work (Ray et al. 2007a), equivalence of three dynamical Λ models viz. $\Lambda \simeq H^2, \Lambda \simeq \ddot{a}/a$ and $\Lambda \simeq \rho$ has been established and in another work (Ray and Mukhopadyay 2007b) age of the Universe is calculated using the same three Λ models. In this work, without taking recourse of any specific model, an overall study of accelerating Universe is done. Importance of Λ for an accelerating Universe is also revealed with special reference to the work of Deb (1999).

Although a large number of dark energy models with both constant and variable Λ are found in the literature, the very phrase 'varying speed of light' itself, in general, has a shuddering effect on mind owing to the panic for collapse of the grand edifice of modern physics through the breakdown of special and general theory of relativity. This situation can be compared with the idea of variation of the gravitational constant G in the pre-relativistic era when Newton's law of Universal Gravitation was the most precious jewel in the kingdom of classical physics. The situation has been reversed after 1905 and at present any idea about a possible change in the speed of light c is considered as a mark of iconoclastic attitude. However, Michell (1784) showed that a particular mass to radius ratio of a star implies that the escape velocity of the star would be equal to the speed of light. But the earliest inception of the idea of changing speed of light before the advent of special theory of relativity is due to Thomson and Tait (1874). The relativistic era has seen many ups and downs regarding the possibility of variation of c. Although Einstein (1911) and Feynman (1988) themselves were not afraid of thinking in terms of a changing speed of light, Eddington (1946) vehemently opposed the idea of a variable c by saying "A variation of c is self-contradictory". In the 1930s, variation of c came into limelight for providing an alternative explanation of cosmological redshift Stewart (1931); Buc (1932); Wold (1935).

But, recent varying speed of light (VSL) theories have a basic difference with the previous ones in the sense that most of them are entangled with the hot Big Bang model of the Universe. The first seminal paper with this cosmological background is due to Moffat (1993). In that (Moffat 1993) and in a subsequent paper (Moffat 2002), Moffat has presented his VSL theory by invoking the idea of phase transition which can solve the horizon problem without taking recourse of the popular idea of inflation. A major problem with the variation of light speed is the violation of energy-conservation law of special theory of relativity, viz. $E = mc^2$, where c is a constant although alternative derivations of this relation without making relativistic idea was shown to be possible (Poincare 1900; Born 1962). But, this problem can be avoided if in Einstein field equations

$$G_{ij} = \kappa T_{ij},\tag{1}$$

(where as weak field approximation κ comes out as $8\pi G/c^4$) only the energy component of T_{ij} is taken as mc^2 then equation (1) comes out as

$$G_{ij} = \frac{8\pi Gm}{c^2}.$$
(2)

The dimensionality of above equation (2) can be accounted for through the component *time-time* of the Einstein field equation for the tensor energy-momentum of fluid perfect. It is clear from this equation (2) that c is inversely proportional to the curvature of space-time and for any departure from $\Omega \neq 1$, the speed of light and the curvature term will adjust themselves so as to make the Universe flat. So, from this point of view, the variation in the speed of light is essential.

After observational evidence in favour of the redshift dependence of the fine structure constant α is established (Web et al. 1999, 2001; Murphy et al. 2001; Web et al. 2003), VSL theories from various point of view has come up. Since α is related to the velocity of light c through the relation $\alpha = e^2/hc$ (where e is the charge of a proton and h is Planck's constant), variability of α implies variability of c provided constancy of e and h is assumed. In dilaton theories, variation of c is assumed (Bekenstein 1982; Barrow et al. 2001; Olive and Pospelov 2001; Sandvik et al. 2002; Bekenstein 2002; Martins 2002; Uzan 2003), but VSL theories pinpoint on the variation of c (Peres 1967; Barrow and Magueijo 1998, 1999; Albrecht and Magueijo 1999; Barrow and Magueijo 2000; Moffat 2002; Peres 2002) although in some cases (Magueijo 2000) h is thought to be responsible for the variability of α . So, it is quite natural to investigate the possible variation of c in the context of the present accelerating Universe, discovered through SN Ia observations (Perlmutter et al. 1998; Riess et al. 1998). In one such work, Camare et al. (2007) have investigated the behaviour of two time-varying models of c, viz. $c(t) \propto a^{-r}$ and $c(t) \propto H^u$ where a is the scale factor and H is the Hubble parameter. The present work is motivated by an intention to investigate analytically the behaviour of a time-varying model of c.

In the first portion of the present investigation, without taking recourse of any specific model, an overall study of accelerating Universe has been done. Importance of dynamic Λ for an accelerating Universe is also revealed with special reference to the work of Deb (1999). In the second part, the question of constancy and variability of the speed of light has been dealt in the framework of the present accelerating Universe by including a time-dependent Λ model in the field equations. In various subsections of Sections 2 and 3, the role of Λ has been demonstrated without resorting to any specific Λ model. In Secs. 4 and 5, respectively Einstein's field equations for variable c have been solved analytically by choosing a particular time-dependent model of Λ and some salient features of the solution have been discussed. Finally, some specific conclusions arrived at from the present work have been presented in Sec. 6.

2 Field Equations with Constant Speed of Light

We know that Einstein's field equation (including cosmological parameter Λ) are given by

$$R^{ij} - \frac{1}{2}Rg^{ij} = -8\pi G \left[T^{ij} - \frac{\Lambda}{8\pi G}g^{ij}\right],\tag{3}$$

where the speed of light c = 1 in relativistic unit and hence is a constant quantity. For the spherically symmetric FLRW metric the above equation yield respectively Friedmann equation and Raychaudhuri equation given by

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G\rho}{3} + \frac{\Lambda}{3},\tag{4}$$

$$\left(\frac{\ddot{a}}{a}\right) = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3} \tag{5}$$

where $\Lambda = \Lambda(t)$ is time-dependent, k is the curvature constant and a(t) is the scale factor of the Universe.

From the equation (4) we have

$$\rho = \frac{3}{8\pi G} \left(\frac{k}{a^2} + H^2 - \frac{\Lambda}{3} \right). \tag{6}$$

So, the present energy density is given by

$$\rho_0 = \frac{3}{8\pi G} \left(\frac{k}{a_0^2} + H_0^2 - \frac{\Lambda_0}{3} \right)$$
(7)

where the suffix zero indicates the present values of the corresponding cosmological parameters. Again, the deceleration parameter q is given by

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -\frac{1}{H^2} \left(\frac{\ddot{a}}{a}\right). \tag{8}$$

Therefore,

$$\frac{\ddot{a}}{a} = -qH^2. \tag{9}$$

Using the equations (9) and (6), we get from (5),

$$-qH^{2} = -\frac{1}{2}\left(\frac{k}{a^{2}} + H^{2} - \frac{\Lambda}{3}\right) - 4\pi Gp + \frac{\Lambda}{3}$$
(10)

which, after simplification, provides the expression for pressure given by

$$p = -\frac{1}{8\pi G} \left[\frac{k}{a^2} + (1 - 2q)H^2 - \Lambda \right].$$
 (11)

Thus, the present value for the pressure is given by

$$p_0 = -\frac{1}{8\pi G} \left[\frac{k}{a_0^2} + (1 - 2q_0)H_0^2 - \Lambda_0 \right].$$
 (12)

Let us choose the barotropic equation of state

$$p = \omega \rho \tag{13}$$

where ω is the barotropic index or equation of state parameter such that $\omega = p/\rho$. In general, ω is a function of time, scale factor or redshift but sometimes it is convenient to consider ω as a constant quantity because current observational data has limited power to distinguish between a time varying and constant equation of state (Kujat 2002; Bartelmann et al. 2005). However, we shall see in Sec. 3.3 that assumption of ω as constant in time will provide interesting physical scenario.

For flat (k = 0) Universe, using equations (6) and (11) in (13), we get

$$\omega = \frac{3\Omega_{\Lambda} + 2q - 1}{3(1 - \Omega_{\Lambda})} \tag{14}$$

where $\Omega_{\Lambda} = \Lambda/3H^2$ is the vacuum energy density of the Universe.

Similarly, for flat Universe, equations (7) and (12) can respectively be written as

$$\rho_0 = \frac{3H_0^2}{8\pi G} (1 - \Omega_{\Lambda_0}) \tag{15}$$

and

$$p_0 = -\frac{H_0^2}{8\pi G} [(1 - 2q_0) - 3\Omega_{\Lambda_0}]$$
(16)

where Ω_{Λ_0} is the present value of the vacuum energy density.

From equations (14), (15) and (16) it is clear that the fate of the Universe depends on q, Ω_{Λ} and H (Fig. 1). Also it is clear from equation (13) that for physical reality Ω_{Λ_0} must be less than one. We also know that the case $p_0 > 0$ provides a collapsing Universe. So, equation (16) tells us that for a collapsing Universe, we must have $(1 - 2q_0) < 3\Omega_{\Lambda_0}$.

3 Physical Features of the Model

3.1 Calculation of ρ_c , ρ_0 and ρ_G

An important quantity which determines the future of the Universe is the critical density ρ_c . The Universe is open or closed according as the present density ρ_0 of the Universe is less or greater than the critical density. Since at present, the accepted value of the Hubble parameter H_0 is $(72 \pm 8) \ km s^{-1} Mpc^{-1}$ (Kirshner 2003), we may choose $H_0 = 72 \ km s^{-1} Mpc^{-1}$ for our calculation. For this value of H_0 , we get $\rho_c = 3H_0^2/8\pi G \sim 9 \times 10^{-30} \ gcm^{-3}$.

In one of our previous work (Ray et al. 2007a) we have shown that the three kinematical Λ models, viz. $\Lambda \sim (\dot{a}/a)^2$, $\Lambda \sim (\ddot{a}/a)$ and $\Lambda \sim \rho$ are equivalent and for these models, $\rho_0 = 3 \times 10^{-30} \ gcm^{-3}$. Also, the measured galactic mass density ρ_G is given by Deb (1999) $\rho_G = 3.1 \times 10^{-31} \ gcm^{-3}$. Therefore,

$$\frac{\rho_G}{\rho_c} \sim 0.033. \tag{17}$$

Also,

$$\frac{\rho_0}{\rho_c} = \frac{1}{3}.\tag{18}$$

From equations (17) and (18) we, immediately, obtain

$$\rho_G \sim 0.1 \rho_0. \tag{19}$$

It is clear from equation (19) that galactic mass density is about 10% of the total mass density of the present Universe. Hence, there must be some hidden mass. Also, equation (18) implies that the present total density of the Universe is one-third of the critical density. This means that galactic (luminous) mass-density is about 3% of the critical density.

Again, the equation (7) can be written as

$$\frac{k}{a^2} = \frac{1}{3} \left[\Lambda_0 - 8\pi G(\rho_c - \rho_0) \right].$$
(20)

From equation (18) it is easy to see that $\rho_0 < \rho_c$ and hence $(\rho_c - \rho_0) > 0$. Also, present observational results indicate that, the Universe is flat (k=0). So, for a flat Universe we must have

$$\Lambda_0 = 8\pi G(\rho_c - \rho_0). \tag{21}$$

On the other hand, for a closed Universe, $\Lambda_0 > 8\pi G(\rho_c - \rho_0)$ whereas for an open Universe, $\Lambda_0 < 8\pi G(\rho_c - \rho_0)$. So, the cosmological parameter is an important factor for determining the geometry of the Universe. Now, one of the predictions of the inflationary theory is a flat Universe and Λ had a large value in the early stages of the Universe. So, one may argue that it is the cosmological parameter which made the Universe flat during inflation.

3.2 Calculation of q_0

For pressureless non-relativistic matter, p = 0. Then from equation (12) we have,

$$\frac{k}{a^2} = \Lambda_0 - (1 - 2q_0)H_0^2.$$
(22)

Using equation (22) in (7), we get

$$\rho_0 = \frac{1}{4\pi G} (\Lambda_0 + 3q_0 H_0^2). \tag{23}$$

Therefore,

$$\frac{\rho_0}{\rho_c} = 2\left(q_0 + \frac{\Lambda_0}{3H_0^2}\right) = 2(q_0 + \Omega_{\Lambda_0}).$$
(24)

Using equation (18) and noting that Ω_{Λ_0} is nearly equal to 0.7 we get from equation (22) that q_0 is about -0.53. This value of q_0 is in excellent agreement with the present accepted value of this parameter (Sahni and Starobinsky 2000) and represents an accelerating Universe. Moreover, Deb (1999), without taking into account the cosmological parameter Λ , showed that q_0 must be positive (equation (6) of Deb (1999)), whereas inclusion of Λ has presented us a situation in which we can suggest that q must be negative. This indicates the inconsistency of the result of Deb (1999) and the present work can be regarded as an improvement over that so far as the present status of the Universe is concerned.

It has been mentioned in the introduction that the present cosmic acceleration has started only recently (a few Gyr. earlier). Before this accelerating phase, the Universe was expanding with deceleration. So, at the turnover stage (from deceleration to acceleration), the deceleration parameter q must have changed its sign. Let us try to find out this signature flip of q. Now, equation (8) can be rewritten as

$$q = -\left(1 + \frac{\dot{H}}{H^2}\right). \tag{25}$$

If we assume $\Lambda \simeq H^2$ then (25) reduces to

$$q = -\left(1 + \frac{\dot{\Lambda}}{2CH^3}\right) \tag{26}$$

where C is a constant. The above equation (26) tells that when $\dot{\Lambda}$ is zero or Λ is a constant, then the Universe always accelerates with a constant acceleration. It may be mentioned here that, by abandoning Λ , Einstein obtained an expanding Universe while the same expanding Universe was obtained by de Sitter for constant Λ . But, when $\dot{\Lambda}/H^3$ is a function of time, then with a proper choice of the constant C, a signature flip of q can be obtained via equation (26).

3.3 Calculation of ω

It is mentioned earlier that, as a simplest case it is useful to model dark energy cosmology with a constant equation of state parameter ω (Kujat 2002; Bartelmann et al. 2005). However, some useful limits on ω was suggested by SNIa data, $-1.67 < \omega < -0.62$ (Knop et al. 2003) whereas refined values come from combined SNIa data with CMB anisotropy and galaxy clustering statistics which is $-1.33 < \omega < -0.79$ (Tegmark et al. 2004). Therefore, let us calculate the value of ω as governed by the value of q_0 and Ω_{Λ_0} in the present case. Putting $q_0 = -0.53$ and $\Omega_{\Lambda} = 0.7$ in equation (12), we get $\omega = 0.044$. So, if $q_0 = -0.53$, then $\omega > 0$ and hence p > 0. Thus, the present accelerating Universe may re-collapse in future if the present value of the deceleration parameter is not greater than -0.53. If $q_0 = -0.55$, then we get p = 0 and hence a dust-filled Universe. It should be also noted here that for quintessence, vacuum fluid and phantom energy, the rate of acceleration should be higher. For instance, when $\omega = -0.5, -1.0$ and -2.0 (note that for their simulations Kuhlen et al. (2005) consider a range of parameter space: $\omega = -0.5, -0.75, -1.0, -1.25$ and -1.5) then we get respectively, $q_0 = -0.775$, -1.0 and -1.45. On the other hand, for stiff fluid ($\omega = 1.0$), $q_0 = -0.1$. So, more smaller the value of ω , higher is the rate of acceleration. This higher acceleration may produce the so-called Big Rip (Caldwell et al. 2003) or Partial Rip (Stefančić 2004) scenario due to divergence of scale factor. Figure 1 shows that throughout the evolution of the Universe, the EOS parameter has assumed negative as well as positive values including the particular value 1 (stiff fluid). Moreover, according to the figure, ω assumes the value 1 when Ω_{Λ} lies between 0.6 and 0.7. So, in the present paper, through an indirect approach, it has been possible to arrive at the two interesting physical ideas of modern cosmology mentioned above. An investigation with a time-dependent ω may reveal more interesting features.

4 Field Equations with Variable Speed of Light

The field equations with varying c are given by

$$3H^2 = 8\pi G\rho + \Lambda c^2 - 3\frac{kc^2}{a^2},$$
(27)

$$2\frac{\ddot{aa}}{a^2} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{kc^2}{a^2} - \Lambda c^2 = -\frac{8\pi Gp}{c^2}.$$
 (28)

For flat Universe, k = 0 and hence, equations (2) and (3) respectively reduce to

$$3H^2 = 8\pi G\rho + \Lambda c^2, \tag{29}$$

$$2(\dot{H} + H^2) + H^2 - \Lambda c^2 = -\frac{8\pi Gp}{c^2}.$$
 (30)

Equation of state is

$$p = \omega \rho c^2, \tag{31}$$

where ω is the barotropic index.

Let us use the *ansatz*

$$\Lambda = \alpha H^2, \tag{32}$$

where α is a parameter.

Then, by using (31) and (32) in (30), we get

$$2\dot{H} + 3H^2 - \alpha c^2 H^2 = -8\pi G \omega \rho. \tag{33}$$

From (29) we get

 $(3 - \alpha c^2)H^2 = 8\pi G\rho.$ (34)

By using (34) in (33), we obtain

$$2\dot{H} = -(3 - \alpha c^2)(1 + \omega)H^2$$
(35)

Suppose $c \propto H^{-1}$, then

$$c = \frac{\beta}{H},\tag{36}$$

where β is a constant of variation. Here we have assumed a time-dependence of velocity of light quanta photon such that $c = c(t) \propto t$. This *ad hoc* assumption helps us to solve the differential equation (35) easily and also yields interesting cosmological scenarios as can be seen later on.

Hence the equation (35) becomes

$$2\dot{H} = \frac{1}{H^2} (\alpha \beta^2 - 3H^2)(1+\omega)H^2.$$
(37)

For pressureless dust, $\omega = 0$ and hence equation (37) reduces to

$$2\dot{H} = (\alpha\beta^2 - 3H^2).$$
(38)

By solving equation (38), we get our solution set as

$$a(t) = a_0 \left[\cosh \sqrt{\frac{3\alpha\beta^2}{4}} t \right]^{\frac{2}{3}},\tag{39}$$

 a_0 is integration constant. It can be readily observed that at cosmological time t = 0 the scale factor becomes $a(t) = a_0$. This means that the present phenomenological Λ -dark energy model is singularity free.

The other solutions for the different physical parameters are

$$H(t) = \sqrt{\frac{\alpha\beta^2}{3}} \left[\tanh \sqrt{\frac{3\alpha\beta^2}{4}} t \right], \qquad (40)$$

$$\Lambda(t) = \frac{\alpha^2 \beta^2}{3} \left[\tanh^2 \sqrt{\frac{3\alpha\beta^2}{4}} t \right], \qquad (41)$$

$$c(t) = \sqrt{\frac{3}{\alpha}} \left[\coth \sqrt{\frac{3\alpha\beta^2}{4}} t \right], \qquad (42)$$

$$\Omega_m = 1 - \left[\coth \sqrt{\frac{3\alpha\beta^2}{4}} t \right]^2, \tag{43}$$

$$\Omega_{\Lambda} = \frac{\alpha}{3}.\tag{44}$$

5 Physical Features of the Model

q

As can be seen, from the above solutions, we have

$$\Omega_m + \Omega_\Lambda = 1 + \frac{\alpha}{3} - \left[\coth \sqrt{\frac{3\alpha\beta^2}{4}t} \right]^2.$$
(45)

Then it is easy to see that as t tends to infinity, the sum of matter and dark-energy densities approaches $\alpha/3$. For the present Universe, $\Omega_m + \Omega_{\Lambda} \simeq 1$ and hence $\alpha \simeq 3$.

Also, the value of the deceleration parameter q is given by

$$= -\left[1 + \frac{3}{2\sinh\sqrt{\frac{3\alpha\beta^2}{4}t}}\right].$$
(46)

For physical validity, both α and β must be nonnegative. Moreover, equation (41) shows that Λ is always positive irrespective of the values of the parameters α and β . This result is consistent with the present accelerating Universe where a positive Λ acts as a repulsive force to generate the acceleration. Since the quantity within the bracket in the expression for q is clearly positive, so q is always negative. Thus, we are getting an ever-accelerating Universe without any signature flipping of q unlike the modern accepted model in which the Universe was decelerating in the past and is accelerating at present (Fig. 2). But, that signature flipping has been obtained already for a number of variable Λ models of phenomenological character with constant c (Ray et al. 2009; Mukhopadhyay et al. 2010). It can be observed from the Fig. 3 that at the early stage of the evolution of the Universe, the velocity of light was greater than the present constant value. However, at the late stages, the value become exactly the

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Fig. 1 The variation of equation of state parameter ω with respect to the vacuum energy density of the Universe (Ω_{Λ}) and the deceleration parameter (q) of the Universe.



Fig. 2 The variation of the deceleration parameter q with respect to t for various values of β and fixed $\alpha = 3.01$.

present accepted value. However, this affirmation is valid only if Eq. (36) can be written in the following form: $\beta = c_0 H_0$, where c_0 would be the present speed of light. This implies that variation in velocity of light is not permitted for phenomenological variable Λ models as also reported elsewhere by Ghosh et al. (2012).

6 Conclusions

The cosmological term Λ and the velocity of light c are two important quantities in the field equations of Einstein. In the present work, their specific roles have been studied for an Universe where dark energy and dark matter are two major constituents. Though variability of Λ is favoured by many workers for avoiding the cosmological constant problem, coincidence problem etc., yet the case of constant Λ cannot be entirely ruled out. Hence, in the first part of the paper, the importance of inclusion of the Λ term in the field equations has been demonstrated in the context of the present accelerating Universe without resorting to any particular Λ model with special reference to the work of Deb (1999).

In the next part of the present work, by choosing a well known time-dependent Λ model, viz. $\Lambda \propto H^2$, it has been shown that the chosen model does not permit the variability of the speed of light. Moreover, it has been already mentioned in the Introduction that the equivalence of the chosen model with other two phenomenological Λ models have been shown by Ray et al. (2007a). This means that those two models also will not permit any kind of change in the speed of light. Apart from arriving at some specific conclusions regarding Λ and c, the case of signature flipping of the deceleration parameter q and the crucial role of the equation of state parameter ω have also been discussed. These are two other salient features of the present work.

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Fig. 3 The variation of the velocity of light c(t) with respect to t for various values of β and fixed $\alpha = 3.01$.

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