

Evolution of Primordial Black Holes in Loop Quantum Gravity

D. Dwivedee^a, B. Nayak^a, M. Jamil^b and L. P. Singh^a

^a*Department of Physics, Utkal University, Vanivihar, Bhubaneswar 751004, India*

^b*Center for Advanced Mathematics and Physics (CAMP),
National University of Sciences and Technology (NUST), H-12, Islamabad, Pakistan*

*E-mail: debabrata@iopb.res.in, bibeka@iopb.res.in,
mjamil@camp.nust.edu.pk, lambodar_uu@yahoo.co.in*

In this work, we study the evolution of Primordial Black Holes within the context of Loop Quantum Gravity. First we calculate the scale factor and energy density of the universe for different cosmic era and then taking these as inputs we study evolution of primordial black holes. From our estimation it is found that accretion of radiation does not affect evolution of primordial black holes in loop quantum gravity even though a larger number of primordial black holes may form in early universe in comparison with Einstein's or scalar-tensor theories.

Keywords: Primordial black holes, loop quantum gravity, accretion, Hawking evaporation

Contents

I. INTRODUCTION	1
II. SOLUTION OF FRIEDMANN EQUATIONS	2
III. ACCRETION OF RADIATION	3
IV. EVAPORATION OF PBH	3
V. PBH DYNAMICS IN DIFFERENT ERA	4
VI. CONSTRAINTS ON PBH	5
VII. CONCLUSION	6
ACKNOWLEDGEMENT	7
References	7

I. INTRODUCTION

The demand for consistency between a quantum description of matter and a geometric description of space-time indicate the necessity of a complete theory of quantum gravity. This theory is expected to provide a new light on singularities present in classical cosmology. Einstein's theory of general relativity leads to the occurrence of space-time singularities in a generic way. So, one may say, General Relativity is severely incomplete and is unable to predict what will come out of a singularity. One of the outstanding problems in classical Einstein cosmology is the Big-Bang singularity which is expected to be solved by quantum gravity. Loop Quantum Gravity (LQG) [1, 2] is one of the best motivated theories of quantum gravity. LQG is a background independent, non perturbative approach to quantum gravity. When Loop Quantum Gravity is applied to cosmology to analyse our universe, it is called Loop Quantum Cosmology(LQC) [3] (also see [4] for a comprehensive review on LQC). In loop quantum cosmology, the non perturbative effects add a term of $-\rho^2/\rho_c$ to the standard Friedmann equation [3, 5], where ρ represents the energy density of the universe and ρ_c is the critical density at which the universe is completely filled with a free massless scalar field when the scale factor reaches a minimum. The modification becomes important when the energy density of the universe approaches critical density(ρ_c) and causes the quantum bounce. So the classical big bang is replaced by a quantum big bounce in such a quantum theory of gravity. Recently more and more researchers have paid their attention to LQC inspired by its appealing features, like avoidance of various singularities [6], inflation in loop quantum cosmology(LQC) [7], large scale effect [8], present cosmic acceleration [9] and so on.

Primordial Black Holes(PBHs) are the black holes formed in the early universe [10]. A comparison of the cosmological density at any time after the Big Bang with the density associated with a black hole shows that PBHs would have mass of the order of the particle horizon mass at their formation epoch. Thus PBHs could span enormous mass range starting from 10^{-5} g to the typical values of 10^{15} g. These black holes could be formed due to initial inhomogeneities [11, 12], inflation [13, 14], phase transitions [15], bubble collisions [16, 17] or the decay of cosmic loops [18]. In 1974 Hawking discovered that the black holes emit thermal radiation due to quantum effects [19]. So the black holes get evaporated depending upon their initial masses. Smaller the initial masses of the PBHs, quicker they evaporate. But the density of a black hole varies inversely with its mass. So high density is required to form lighter black holes and such high densities are available only in the early universe. So primordial black holes are the only black holes whose masses could be small enough to have evaporated by present time. There have been speculations that PBHs could act as seeds for structure formation [20] and could also form a significant component of dark matter [21]. Since the cosmological environment was very hot and dense in the radiation dominated era, an appreciable amount of energy-matter from the surroundings can be absorbed by black holes. Such accretion is responsible for the prolongation of life time of PBHs [22, 23].

In this work, we study the evolution of PBH within the context of loop quantum gravity. First, we estimate the cosmic scale factor $a(t)$ and energy density $\rho(t)$ of the fluid filling the universe for different cosmic era within the context of loop quantum gravity. Taking these as inputs, PBH evolution is studied considering both the Hawking evaporation and accretion of radiation by the PBH. The primary aim being to compare the results so obtained with the analyses carried out earlier within the context of General Theory of Relativity and Brans-Dicke theories.

II. SOLUTION OF FRIEDMANN EQUATIONS

For a spatially flat FRW universe($k=0$) with scale factor(a), two Friedmann equations in loop quantum gravity [3], take the form :

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{8\pi G}{3}\rho\left(1 - \frac{\rho}{\rho_c}\right) \quad (1)$$

$$\dot{H} = -\frac{4\pi G}{3}(\rho + p)\left(1 - \frac{2\rho}{\rho_c}\right) \quad (2)$$

where H is the Hubble parameter, ρ is the energy density and P is the pressure of the fluid filling the universe.

The energy conservation equation is given by

$$\dot{\rho} + 3H(\rho + p) = 0 \quad (3)$$

From energy conservation equation, we get

$$\rho \propto a^{-4} \text{ for } t < t_e \quad (4)$$

$$\rho \propto a^{-3} \text{ for } t > t_e \quad (5)$$

where t_e is the time of radiation-matter equality.

Using this solution in equation(1), one gets the temporal behaviour of the scale factor $a(t)$ as shown below.

For radiation dominated era ($t < t_e$) :

$$a(t) = \left[\frac{\rho_0 a_0^3 a_e}{\rho_c} + \left\{ 2\rho_0^{1/2} a_0^{3/2} a_e^{1/2} \sqrt{\frac{8\pi G}{3}}(t - t_e) + \left(a_e^4 - \frac{\rho_0 a_0^3 a_e}{\rho_c} \right)^{1/2} \right\}^2 \right]^{1/4} \quad (6)$$

where the subscript 0 indicates the present value of any parameter and $a_e = a(t_e)$ and ρ_c represents the critical value of energy density of the universe given by $\rho_c = \frac{\sqrt{3}}{16\pi^2 \gamma^3} \rho_{pl}$ with $\gamma = \frac{\ln 2}{\pi\sqrt{3}}$ is the dimensionless Barbero-Immirzi parameter [24] and ρ_{pl} is the energy density of universe in Planck time.

For matter dominated era ($t > t_e$) :

$$a(t) = \left[\frac{\rho_0 a_0^3}{\rho_c} + \left\{ \frac{3}{2} \rho_0^{1/2} a_0^{3/2} \sqrt{\frac{8\pi G}{3}}(t - t_0) + \left(a_0^3 - \frac{\rho_0 a_0^3}{\rho_c} \right)^{1/2} \right\}^2 \right]^{1/3} \quad (7)$$

Using equations (4) and (6), we get

For $t < t_e$

$$\rho(t) = \rho_0 \left[\frac{\rho_0}{\rho_c} + \left\{ 2\sqrt{\frac{8\pi G}{3}} \rho_0^{1/2} (t - t_e) + \frac{3}{2} \sqrt{\frac{8\pi G}{3}} \rho_0^{1/2} (t_e - t_0) + \left(1 - \frac{\rho_0}{\rho_c} \right)^{1/2} \right\}^2 \right]^{-1} \quad (8)$$

Using equations (5) and (7), we get

For $t > t_e$

$$\rho(t) = \rho_0 \left[\frac{\rho_0}{\rho_c} + \left\{ \frac{3}{2} \sqrt{\frac{8\pi G}{3}} \rho_0^{1/2} (t - t_0) + \left(1 - \frac{\rho_0}{\rho_c} \right)^{1/2} \right\}^2 \right]^{-1} \quad (9)$$

III. ACCRETION OF RADIATION

When a PBH evolves through radiation dominated era, it can also accrete radiation from the surrounding. The accretion of radiation leads to increase of its mass with the rate given by

$$\dot{M}_{acc} = 4\pi f r_{BH}^2 \rho_r \quad (10)$$

where ρ_r is the radiation energy density of the surrounding of the black hole, r_{BH} is the black hole radius and f is the accretion efficiency. The value of the accretion efficiency f depends on the complex physical processes such as the mean free path of the particles comprising the radiation surrounding PBHs. Any peculiar velocity of the PBH with respect to the cosmic frame could increase the value of f [25, 26]. Since the precise value of f is unknown, it is customary [27] to take the accretion rate to be proportional to the product of the surface area of the PBH and the energy density of radiation with $f \sim O(1)$.

After substituting the expressions for $r_{BH} = 2GM$ and ρ_r given by equation(8) in equation (10) , we get

$$\dot{M}_{acc} = 16\pi f G^2 M^2 \rho_0 \left[\frac{\rho_0}{\rho_c} + \left\{ 2\sqrt{\frac{8\pi G}{3}} \rho_0^{1/2} (t - t_e) + \frac{3}{2} \sqrt{\frac{8\pi G}{3}} \rho_0^{1/2} (t_e - t_0) + \left(1 - \frac{\rho_0}{\rho_c} \right)^{1/2} \right\}^2 \right]^{-1} \quad (11)$$

Solving equation(11), one can find

$$M(t) = \left\{ M_i^{-1} - 8\pi f G^2 \rho_c^{1/2} \sqrt{\frac{3}{8\pi G}} \tan^{-1} \left[\frac{\rho_c}{\rho_0} Z(t) - 1 \right]^{1/2} + 8\pi f G^2 \rho_c^{1/2} \sqrt{\frac{3}{8\pi G}} \tan^{-1} \left[\frac{\rho_c}{\rho_0} Z(t_i) - 1 \right]^{1/2} \right\}^{-1} \quad (12)$$

Again using horizon mass as initial mass of PBH i.e; $M_i = M_H(t_i) = G^{-1} t_i$, we get

$$M(t) = M_i \left[1 - 8\pi f G^{1/2} \rho_c^{1/2} t_i \sqrt{\frac{3}{8\pi}} \{ \tan^{-1} \left[\frac{\rho_c}{\rho_0} Z(t) - 1 \right]^{1/2} - \tan^{-1} \left[\frac{\rho_c}{\rho_0} Z(t_i) - 1 \right]^{1/2} \} \right]^{-1} \quad (13)$$

where $Z(t) = \frac{\rho_0}{\rho_c} + \left\{ 2\sqrt{\frac{8\pi G}{3}} \rho_0^{1/2} (t - t_e) + \frac{3}{2} \sqrt{\frac{8\pi G}{3}} \rho_0^{1/2} (t_e - t_0) + \left(1 - \frac{\rho_0}{\rho_c} \right)^{1/2} \right\}^2$.

IV. EVAPORATION OF PBH

As is well known black holes can also loose mass through Hawking evaporation. The rate at which the PBH mass decreases due to evaporation is given by

$$\dot{M}_{evap} = -4\pi r_{BH}^2 a_H T_{BH}^4 \quad (14)$$

where $a_H \sim$ is the Blackbody Constant and $T_{BH} \sim$ is the Hawking Temperature $= \frac{1}{8\pi GM}$
Now equation(14) becomes

$$\dot{M}_{evap} = -\frac{a_H}{256\pi^3} \frac{1}{G^2 M^2} \quad (15)$$

Integrating the above equation, we get

$$M = \left[M_i^3 + 3\alpha(t_i - t) \right]^{1/3} \quad (16)$$

where $\alpha = \frac{a_H}{256\pi^3} \frac{1}{G^2}$

and M_i is the the black hole mass at its formation time t_i .

We rewrite equation (16) as

$$M(t) = M_i \left[1 + \frac{3\alpha}{M_i^3} (t_i - t) \right]^{1/3} \quad (17)$$

V. PBH DYNAMICS IN DIFFERENT ERA

Primordial Black Holes, as discussed earlier, are formed only in radiation dominated era. We now study PBHs so formed in two categories:

- (i) PBHs evaporating in radiation dominated era ($t_{evap} < t_e$)
- (ii) PBHs evaporating in matter dominated era ($t_{evap} > t_e$)

CASE-1 ($t_{evap} < t_e$)

If we consider both evaporation and accretion simultaneously, then the rate at which primordial black hole mass changes is given by

$$\begin{aligned} \dot{M}_{PBH} = & 16\pi f G^2 M^2 \rho_0 \left[\frac{\rho_0}{\rho_c} + \left\{ 2\sqrt{\frac{8\pi G}{3}} \rho_0^{1/2} (t - t_e) \right. \right. \\ & \left. \left. + \frac{3}{2}\sqrt{\frac{8\pi G}{3}} \rho_0^{1/2} (t_e - t_0) + \left(1 - \frac{\rho_0}{\rho_c} \right)^{1/2} \right\}^2 \right]^{-1} - \frac{a_H}{256\pi^3} \frac{1}{G^2 M^2} \end{aligned} \quad (18)$$

Solving above equation numerically, we construct the following table for PBHs which are evaporating in radiation dominated era. In our calculation, we have used $\rho_0 = 1.1 \times 10^{-29} g - cm^{-3}$, $\rho_c = 5.317 \times 10^{94} g - cm^{-3}$, $G = 6.673 \times 10^{-8} dyne - cm^2/g^2$, $t_e = 10^{11} s$, $t_0 = 4.42 \times 10^{17} s$ and $M_e = 10^{49} g$.

t_i	M_i	t_{evap}	
		$f = 0$	$f = 1$
$10^{-32} s$	$10^6 g$	$3.333 \times 10^{-11} s$	$3.333 \times 10^{-11} s$
$10^{-30} s$	$10^8 g$	$3.333 \times 10^{-5} s$	$3.333 \times 10^{-5} s$
$10^{-28} s$	$10^{10} g$	$3.333 \times 10^1 s$	$3.333 \times 10^1 s$
$10^{-26} s$	$10^{12} g$	$3.333 \times 10^7 s$	$3.333 \times 10^7 s$

TABLE I: Display of formation times and initial masses of the PBHs evaporating in radiation dominated era

It is clear from Table-1 that with increase in initial mass, evaporating time increases. However radiation accretion, surprisingly, seems to have little effect on evolution of PBH unlike the results obtained in theories of Einstein or scalar-tensor type. This is also shown in Figure-1.

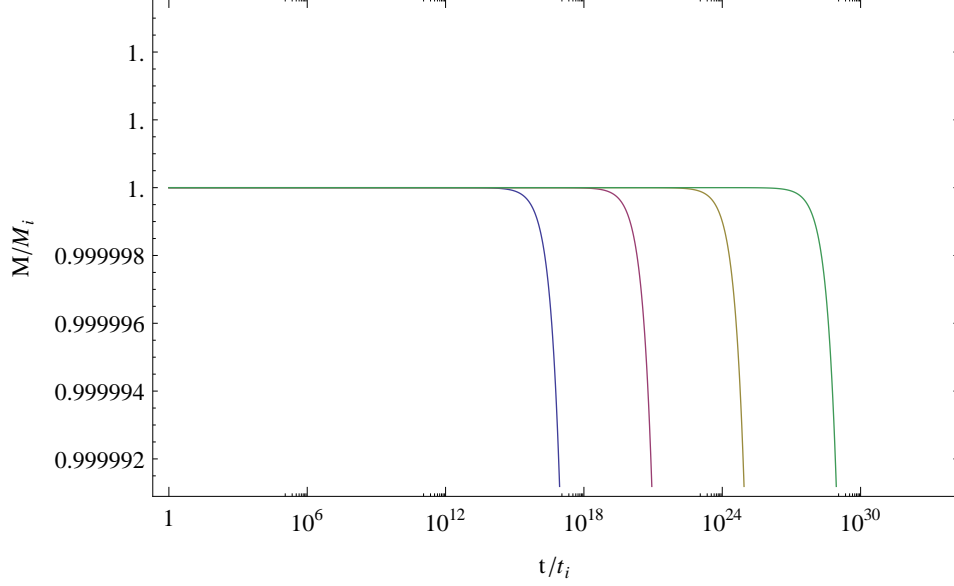


FIG. 1: Evaporation of PBHs for different initial masses (i.e. $10^6 g$, $10^8 g$, $10^{10} g$ and $10^{12} g$) are shown in the Figure where axes are taken in logarithmic scale

CASE-II ($t_{evap} > t_e$)

Since there is insignificant accretion of radiation in matter dominated era, the first term in the combined equation(18) for variation of M_{PBH} with time should be integrated only upto t_e . Basing upon the numerical solution we construct Table-2 for the PBHs evaporating by the present time.

$t_{evap} = t_0 = 4.42 \times 10^{17} s$		
f	t_i	M_i
0	$2.3669 \times 10^{-23} s$	$2.3669 \times 10^{15} g$
0.25	$2.3669 \times 10^{-23} s$	$2.3669 \times 10^{15} g$
0.5	$2.3669 \times 10^{-23} s$	$2.3669 \times 10^{15} g$
1.0	$2.3669 \times 10^{-23} s$	$2.3669 \times 10^{15} g$

TABLE II: Display of formation times of PBHs which are evaporating now for several accretion efficiencies

It is clear from Table-2 that PBH evaporation is again not affected by radiation accretion efficiency.

VI. CONSTRAINTS ON PBH

The fraction of the Universe mass going into PBHs at time t is [9]

$$\beta(t) = \left[\frac{\Omega_{PBH}(t)}{\Omega_R} \right] (1+z)^{-1} \quad (19)$$

where $\Omega_{PBH}(t)$ is the density parameter associated with PBHs formed at time t , z is the redshift associated with time t and Ω_R is the microwave background density having value 10^{-4} .

Substituting the value of Ω_R in the above equation, we get

$$\beta(t) = (1+z)^{-1} \Omega_{PBH}(t) \times 10^4 \quad (20)$$

For $t < t_e$, redshift definition implies

$$(1+z)^{-1} = \frac{a(t)}{a(t_0)} = \frac{a(t)}{a(t_e)} \frac{a(t_e)}{a(t_0)} \quad (21)$$

But here

$$\frac{a(t)}{a(t_e)} = \left[\frac{\rho_0}{\rho_c} \frac{a_0^3}{a_e^3} + \left\{ 2\rho_0^{1/2} \frac{a_0^{3/2}}{a_e^{3/2}} \sqrt{\frac{8\pi G}{3}} (t - t_e) + \left(1 - \frac{\rho_0}{\rho_c} \frac{a_0^3}{a_e^3} \right)^{1/2} \right\}^2 \right]^{1/4} \quad (22)$$

and

$$\frac{a(t_e)}{a(t_0)} = \left[\frac{\rho_0}{\rho_c} + \left\{ \frac{3}{2} \rho_0^{1/2} \sqrt{\frac{8\pi G}{3}} (t_e - t_0) + \left(1 - \frac{\rho_0}{\rho_c} \right)^{1/2} \right\}^2 \right]^{1/3} \quad (23)$$

Using above numerical values of different quantities in equation(23), we get

$$\frac{a(t_e)}{a(t_0)} = 0.746 \quad (24)$$

Using equations (22) and (24) in equation (21), we get

$$(1+z)^{-1} = \left[\frac{\rho_0}{\rho_c} \frac{a_0^3}{a_e^3} + \left\{ 2\rho_0^{1/2} \frac{a_0^{3/2}}{a_e^{3/2}} \sqrt{\frac{8\pi G}{3}} (t - t_e) + \left(1 - \frac{\rho_0}{\rho_c} \frac{a_0^3}{a_e^3} \right)^{1/2} \right\}^2 \right]^{1/4} \times 0.746 \quad (25)$$

Substituting equation(25) in equation (20), we get

$$\beta(t) = \left[\frac{\rho_0}{\rho_c} \frac{a_0^3}{a_e^3} + \left\{ 2\rho_0^{1/2} \frac{a_0^{3/2}}{a_e^{3/2}} \sqrt{\frac{8\pi G}{3}} (t - t_e) + \left(1 - \frac{\rho_0}{\rho_c} \frac{a_0^3}{a_e^3} \right)^{1/2} \right\}^2 \right]^{1/4} \times 0.746 \times \Omega_{PBH}(t) \times 10^4 \quad (26)$$

Using $M = G^{-1}t$, we can write equation (26) to represent the fraction of the Universe going into PBHs' as a function of mass M as

$$\beta(M) = \left[\frac{\rho_0}{\rho_c} \frac{a_0^3}{a_e^3} + \left\{ 2\rho_0^{1/2} \frac{a_0^{3/2}}{a_e^{3/2}} \sqrt{\frac{8\pi G}{3}} G(M - M_e) + \left(1 - \frac{\rho_0}{\rho_c} \frac{a_0^3}{a_e^3} \right)^{1/2} \right\}^2 \right]^{1/4} \times 0.746 \times \Omega_{PBH}(M) \times 10^4 \quad (27)$$

Observations of the cosmological deceleration parameter imply that $\Omega_{BH}(M) < 1$ over all mass ranges for which PBHs have not evaporated yet. But presently evaporating $PBHs(M_*)$ generate a γ -ray background whose most of the energy is appearing at around 100 Mev [28]. If the fraction of the emitted energy which goes into photons is ϵ_γ , then the density of the radiation at this energy is expected to be $\Omega_\gamma = \epsilon_\gamma \Omega_{PBH}(M_*)$. Since $\epsilon_\gamma \sim 0.1$ [29] and the observed γ -ray background density around 100 Mev is $\Omega_\gamma \sim 10^{-9}$, we get $\Omega_{PBH} < 10^{-8}$.

With the use of all these parameters, Eqn (27) leads to an upper bound

$$\beta(M_*) < 0.746 \times 10^{-4} \quad (28)$$

Here for the presently evaporating PBHs the upper limit is much greater than previous results [23, 30, 31] obtained by assuming Einstein's theory or Brans Dicke theory as the theory of gravity. But from our calculation, we found that the formation time of presently evaporating PBHs is nearly same in all theories. This higher upper bound implies that in LQC a much larger number of PBHs would form in early Universe in comparison with standard cosmology and scalar-tensor theories.

VII. CONCLUSION

We have studied PBH evolution in loop quantum gravity. We have estimated the cosmic scale factor $a(t)$ and energy density $\rho(t)$ of the universe for both radiation dominated era and matter dominated era. Both expressions for $a(t)$ and $\rho(t)$ are different from those in standard cosmology. In the limit $\rho_c \rightarrow \infty$ these expressions go over to those of standard cosmology since standard cosmology envisages nearly zero size for the universe at the time of creation. Using these results as inputs we have studied evolution of PBHs using both accretion of radiation and evaporation. We find accretion of radiation has no effect on PBH evaporation in the present formalism. From numerical calculation it is found that the PBHs created before 1.443×10^{-25} s could evaporate completely in radiation dominated era and the accretion efficiency does not affect the evaporation of individual PBHs formed at different times in this era. Further, we found that the upper bound on initial PBH mass fraction is much greater than all previous analyses but the formation time of presently evaporating PBHs is nearly the same. The greater upperbound implies that a large number of PBHs could possibly form in early universe within the context of LQG in comparison with standard cosmology and scalar-tensor theories.

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