Fundamental properties of lower main-sequence stars

Guillermo Torres*

Harvard-Smithsonian Center for Astrophysics, 60 Garden St., Cambridge MA 02138, USA

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The field of exoplanet research has revitalized interest in M dwarfs, which have become favorite targets of Doppler and transit surveys. Accurate measurements of their basic properties such as masses, radii, and effective temperatures have revealed significant disagreements with predictions from stellar evolution theory in the sense that stars are larger and cooler than expected. These anomalies are believed to be due to high levels of activity in these stars. The evidence for the radius discrepancies has grown over the years as more and more determinations have become available; however, fewer of these studies include accurate determinations of the temperatures. The ubiquitous mass-radius diagrams featured in many new discovery papers are becoming more confusing due to increased scatter, which may be due in part to larger than realized systematic errors affecting many of the published measurements. A discussion of these and other issues is given here from an observer's perspective, along with a summary of theoretical efforts to explain the radius and temperature anomalies.

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1 Introduction

A common justification for studying late-type stars, aside from being interesting objects in themselves, is that they dominate the stellar population in our Galaxy by number: roughly 75% of the points of light in the sky are M dwarfs. In recent years they have also become attractive targets for exoplanet searches (Nutzman et al. 2008; Law et al. 2011a; Sipőcz et al. 2011; Barnes et al. 2012; and others), particularly when looking for small planets in the habitable zones of their parent stars. This has provided extra motivation for studying them. Both the Doppler signals and the transit signals are larger and more easily detectable for planets around M dwarfs, and the lower stellar luminosities mean that orbits in the habitable zone are closer in, making transits more likely and more frequent when they do occur.

An important by-product of recent transit surveys has been the discovery of many eclipsing binaries with M-dwarf components (e.g., Law et al. 2011b; Coughlin et al. 2011; Harrison et al. 2012; Birkby et al. 2012). These kinds of systems have traditionally been the most favorable for determining the basic properties of late-type stars, including their mass, radius, temperature, and luminosity. Unfortunately, however, most newly discovered systems tend to be faint, and the bottleneck for accurate determinations continues to be the spectroscopy.

Masses and radii in double-lined eclipsing binaries can be obtained free of assumptions, as their derivation depends only on Newtonian physics and geometry. Many determinations have been made also in single-lined eclipsing binaries, but these are less fundamental as they require previous knowledge of the primary mass or radius, or the assumption of synchronous rotation of the primary and spin-orbit alignment in circular orbits. Long-baseline interferometry has enabled the measurement of absolute radii for increasing numbers of single late-type stars for which the parallax is known, but masses for these stars cannot be obtained dynamically, so they are typically estimated from a massluminosity relation. These assumptions limit the usefulness of single-lined eclipsing binaries and single stars for testing models of stellar evolution.

2 Measuring fundamental properties

The techniques for measuring absolute masses and radii of stars in eclipsing binaries are straightforward and sufficiently well known that we will dispense with a description here. For details the reader is referred to the reviews by Andersen (1991), or Torres, Andersen & Giménez (2010). The application of these methods still requires care, though, if precisions (and accuracies) of 3% or better are to be obtained, as are generally necessary for meaningful comparisons with stellar evolution theory for low-mass stars. Temperature determinations are less fundamental. In eclipsing binaries they are typically derived either through color indices and empirical calibrations (if the reddening is known). or from the luminosities and radii if the distance is known. Spectroscopic temperature determinations remain difficult due to the complexities of modeling molecular features in the atmospheres of cool stars, not to speak of the fact that the observed spectra in binaries are typically double-lined.

^{*} Corresponding author. e-mail: gtorres@cfa.harvard.edu

Metallicities have also been challenging to determine spectroscopically for similar reasons, although progress is being made in calibrating composition in terms of optical and near-infrared indices (e.g., Woolf et al. 2005, 2006, 2009; Rojas-Ayala et al. 2010, 2012; Reylé et al. 2011). Photometric calibrations using various color indices are also available (Bonfils et al. 2005; Casagrande et al. 2008; Johnson et al. 2009, 2012; Schlaufman et al. 2010).

3 Discrepancies with models

Some of the observed properties of low-mass stars are known to disagree with predictions from standard stellar evolution theory. Most notably, the measured radii for M stars in binaries are larger than indicated by models, typically by 5-10% for the best measured systems, or sometimes more depending on the model. Early indications of this problem for M dwarfs were reported by Hoxie (1970, 1973) and Lacy (1977), and were subsequently supported by additional accurate measurements by Popper (1997), Clausen (1999), and many others. More recent determinations have removed any observational doubt that the models do not fit the observations as well as they should. While this radius problem has received the most attention, the effective temperatures of low-mass stars are also in disagreement with theory, although far fewer studies have shown this because temperatures are more difficult to determine. Real stars tend to be too cool compared to predictions, by about half as much as the radii, in relative terms.

For decades there were only two low-mass eclipsing binaries with properties measured accurately enough to notice the problem: the classical systems CM Dra ($\sim 0.22 M_{\odot}$; see Morales et al. 2009) and YY Gem ($\sim 0.60 M_{\odot}$; Torres & Ribas 2002). Then came CU Cnc ($\sim 0.4 M_{\odot}$; Ribas 2003) and GU Boo ($\sim 0.6 M_{\odot}$; López-Morales & Ribas 2005), and although many others have been published since then, relatively few of these studies can claim realistic uncertainties in the masses and radii under 3%, and show convincingly that systematic errors are under control.

In recent years it has become clear that these discrepancies with the models are not confined to the M dwarfs (Torres et al. 2006; Clausen et al. 2009); some stars as massive as the Sun (or more generally, stars with convective envelopes) also seem to be "inflated" and too cool when compared with models.

The clearest evidence of "radius inflation" and "temperature suppression" is seen by examining the best-studied individual systems. CM Dra is the poster child for these anomalies (see also discussions by Feiden et al. 2011, Spada & Demarque 2012, and MacDonald & Mullan 2012). It is presumed to be a Population II binary based on its extreme kinematics, although its precise age is not known. The metallicity has also been controversial, with recent determinations apparently converging toward a value near [Fe/H] = -0.4 (e.g., Kuznetsov et al. 2012). Figure 1 presents the comparison between the measured radii and



Fig. 1 Measured masses, radii, and effective temperatures for CM Dra compared with isochrones by Dotter et al. (2008). In the lower panel the 4 Gyr and 10 Gyr isochrones at each metallicity are indistinguishable.

temperatures of CM Dra as determined by Morales et al. (2009) against isochrones from the Dartmouth series by Dotter et al. (2008) for two metallicities and two different ages. One may draw two conclusions from this diagram: 1) matching the radii and temperatures requires models with an unusually high metallicity around [Fe/H] = +0.5, which is at odds with expectations; 2) despite claims often seen in the literature, age is not totally irrelevant when comparing slowly-evolving low-mass stars such as these with models. In fact, the figure shows that changing the isochrone age from 4 to 10 Gyr leads to a significant change in the predicted radii compared to the uncertainties.

Similar discrepancies are seen for YY Gem and CU Cnc. These two are particularly important systems because there is some additional (if somewhat circumstantial) information on the age and metallicity that eliminates free parameters when comparing with models. Another of the well-studied systems, GU Boo, has components that are also too large and too cool compared to predictions, and in this case even models with a metallicity as high as [Fe/H] = +0.5 do not quite match the measured radii.

It has long been realized that essentially all of these discrepant low-mass binary systems have short orbital periods, typically less than 3 days. The leading hypothesis to explain the radius inflation and temperature suppression has therefore been that tidal forces in these tight systems tend to synchronize the stellar spins with the orbital motion, resulting in rapid rotation and associated magnetic activity. Activity is known to inhibit convective transport of energy in the stellar interiors, and this in turn leads to larger radii and cooler temperatures. There is at least a first-order theoretical understanding of these processes (e.g., Gough & Taylor 1966; Mullan & MacDonald 2001; Chabrier, Gallardo & Baraffe 2007; MacDonald & Mullan 2012), as well as empirical evidence that the use of a smaller mixing length parameter $\alpha_{\rm ML}$ in the models (to emulate reduced convective efficiency) does indeed improve the fit to the observations. Spot coverage associated with stellar activity reduces the radiating surface area, and this can also contribute to the discrepancies. Alternate explanations, such as errors in the opacities or metallicity effects, seem less likely.

If the short orbital periods are at the root of the problem, then a logical expectation would be that systems with longer periods would agree with the models much better. Tidal forces should be weaker, the stars would presumably not be synchronized and should rotate more slowly, and activity would therefore be much lower. Unfortunately longperiod eclipsing binaries (P > 10 days) are rare and exceedingly difficult to study (it can take several observing seasons, good luck with the weather, and plenty of patience to complete a light curve), and until recently there were no such systems with accurate mass and radius determinations. In 2011 the MEarth and Kepler transit surveys reported the discovery of two eclipsing binaries with low-mass components that coincidentally have the same orbital period (about 41 days) to within 0.1%. Contrary to expectations, the low-mass stars in the MEarth system LSPM J1112+7626 $(0.39 M_{\odot} \text{ and } 0.27 M_{\odot};$ Irwin et al. 2011) both have inflated radii and are cooler than predicted by theory. The secondary in Kepler-16 (0.20 M_{\odot} ; Doyle et al. 2011; Winn et al. 2011) is also inflated. Its temperature has not been determined, and the primary is a more massive K star. The activity level of Kepler-16B is unknown, and there is perhaps some evidence of activity in the secondary of LSPM J1112+7626, even at this long a period.

The situation regarding the nature of the discrepancies with models is thus not as clear-cut as expected, and to make matters more interesting, there is at least one well studied triple system from *Kepler* with two M-dwarf components (KOI-126; Carter et al. 2011) in which the stellar radii seem to agree perfectly with the Dartmouth models (see Feiden et al. 2011).

4 Putting it all in together

The vast numbers of light curves produced by exoplanet transit surveys and the increased interest in these disagreements between theory and observation have led to a surge in low-mass eclipsing binary discoveries in the last few years. It has become routine for each new discovery paper to present a (typically well-populated) mass-radius diagram highlighting the new system, and showing one or another set of model isochrones for comparison. The objects displayed are usually drawn from many different sources, sometimes including interferometric measurements for single stars or results from eclipsing single-lined spectroscopic binaries. Authors have used such diagrams to draw general



Fig. 2 Mass-radius diagram for low-mass stars, including all measurements for double-lined eclipsing binaries (SB2s, filled symbols) as well as determinations for single-lined eclipsing systems (SB1s) and single stars (open symbols). Solar-metallicity Dartmouth isochrones are shown for comparison, for ages ranging from 1 to 13 Gyr (grey band).

conclusions about the magnitude of the radius anomalies, and some studies have even proposed patterns in the discrepancies depending on mass or period.

In a review on low-mass stars a few years ago, Ribas (2006) provided an interesting illustration of the confusion that can result from including M-dwarf systems indiscriminately in such mass-radius diagrams. For a first version of the diagram he compiled and included all low-mass systems known at the time (34 stars, including single-lined binaries and stars with measured angular diameters), and noted that on average the data appeared discrepant with the models above the convective boundary (~0.35 M_{\odot}), but seemed to agree better with theory below that limit. He then restricted the sample to only the double-lined eclipsing systems with mass and radius errors under 3%, and the deviations were seen much more clearly, all the way to the lowest-mass system (CM Dra) below the convective boundary.

Many more determinations have become available since. Figure 2 shows an update of the mass-radius diagram that includes all double-lined as well as single-lined eclipsing systems of which we are aware with at least one component measured to be under $0.7 M_{\odot}$. In also includes single stars in this range with radii determined interferometrically and masses inferred from a mass-luminosity relation. There are a total of 108 stars displayed on the graph. For the vast majority the chemical composition is unknown, so we have chosen to compare the observations with Dartmouth models for solar metallicity. We also do not know the ages of most of these stars, and as indicated earlier age can affect the radius in a non-negligible way even for the slowly-evolving lowest-mass stars. Some authors have compared the obser-





Fig.3 Same as Fig. 2, limited to stars with mass and radius errors under 2% for double-lined eclipsing binaries, and radius errors below that limit for single-lined systems and single stars. No restriction was placed on the mass errors for the later two classes of objects, as the masses are often adopted from a mass-luminosity relation or rely on other assumptions.

vations against a very young isochrone (e.g., 300 Myr), but this is not necessarily a typical age for a field star and will tend to exaggerate the radius discrepancies. Therefore, we have chosen to display isochrones for all ages from 1 to 13 Gyr. The model is thus represented by a band instead of a single line.

The scatter in Figure 2 is so large that very little can be concluded from this diagram, and some may even be tempted to say that there is nothing wrong with the models. This is of course not true, as we have pointed out before. Restricting the sample to only the stars with formal errors under 5% (56 objects) doesn't change the picture significantly. The radius discrepancies are still obscured by the scatter and the uncertainties in the models. In Figure 3 we have been even more selective, setting the error limit to 2% (24 stars). Given the age uncertainties, one is still hard-pressed to draw any meaningful conclusions.

Stars are of course under no obligation to all have solar metallicity. So as an exercise, if we now display models (all ages) for [Fe/H] = -0.5 and [Fe/H] = +0.5, in addition to solar composition, any systematic differences between the isochrones and the observations appear even less obvious (Figure 4).

5 The scatter in the mass-radius diagram

In the last few years it has become increasingly clear (at least to the author) that successive updates of the massradius diagram featuring more and more low-mass stars

Fig. 4 Same as Fig. 3, but now showing model isochrones for metallicities of [Fe/H] = -0.5 and [Fe/H] = +0.5, in addition to solar, and all ages from 1 to 13 Gyr.

have not necessarily led to a deeper understanding of the problem. The scatter in the diagram has become quite large, and the evidence for disagreements between theory and observation, which is unmistakable when focusing on the best studied individual systems, is getting blurred when looking at the larger sample of all available determinations. This loss of clarity from the increased dispersion is due to at least three causes: 1) published formal errors do not necessarily reflect the total uncertainty; in fact, systematic errors tend to dominate in this mass regime, as discussed further below; 2) in the great majority of cases the age and metallicity are unknown; quantifying the radius discrepancy by comparison with an arbitrarily chosen model can be misleading; 3) the degree of radius inflation may not be the same for all stars (recall KOI-126, which shows no such anomaly); it remains to be confirmed whether this is a function of the strength of the activity (which is not always a known property), stellar mass, or some other parameter.

In a recent review on accurate stellar masses and radii, Torres et al. (2010) examined the credentials of all known double-lined eclipsing binaries studied up to that time, and presented a short list of only four M-dwarf systems with well-measured properties (relative errors of 3% or less) that satisfied their strict selection criteria. These four systems are CM Dra, YY Gem, CU Cnc, and GU Boo. Many authors have adopted similar cutoffs for the errors when selecting new (or old) systems for the mass-radius diagram, but have tended to overlook other selection criteria that are perhaps more difficult to apply and require personal judgement, but are just as important. We suspect the masses and radii for many of these often used systems may be biased, and can potentially lead to more confusion. To state the obvious, precision is not the same as accuracy.

In addition to setting an upper limit on the errors, it is important to verify that the quality and quantity of the data used in the mass and radius determinations are adequate to support the claimed uncertainties (see Sect. 3 of Torres et al. 2010, and also Andersen 1991). Incomplete light curves, such as ones with no out-of-eclipse observations or that do not fully cover both eclipses, can easily lead to systematic errors in the radii. Similarly, radial-velocity curves based on only a handful of observations, or with less-thanoptimal phase coverage (e.g., missing one or both quadratures) are also questionable, regardless of the internal precision of the velocities. The data analysis techniques are important as well. Light curves often present strong degeneracies among several of the fitted parameters, and it is all too easy to be misled by the quick convergence of the solutions. In particular, determining the individual radii (or their ratio) accurately in partially eclipsing systems with nearly equal components can be difficult, and in many cases requires the application of external constraints such as a spectroscopic luminosity ratio to remove the degeneracy (see Andersen 1991).

The most reliable analyses are those that pay close attention to all of these issues, and document consistency checks or other efforts to control or at least assess the impact of systematic errors. Although many new mass and radius determinations for low-mass binary systems have appeared in the literature since the compilation of Torres et al. (2010), and are commonly shown in recent mass-radius diagrams, we have a feeling that only a small number meet all of the stringent criteria outlined above. Deciding which studies to trust requires a critical review of the published analyses, a task that is beyond the scope of this paper.

For the above reasons we believe we may have reached the point at which it is more fruitful to compare wellmeasured individual systems against models, rather than to include a larger number of more questionable determinations and attempt to draw general quantitative conclusions.

6 Understanding the anomalies

A number of investigations, both observational and theoretical, have examined possible correlations between the radius and temperature anomalies and the strength of the stellar activity. On the observational side, López-Morales (2007) used the X-ray luminosity as an activity indicator, and showed that for close binary stars with low-mass components the degree of radius inflation seems to depend on $L_{\rm X}/L_{\rm bol}$, while for single stars no clear dependence was seen. As others have pointed out, however, the range of activity levels for the single stars in that study was rather small, so the conclusion should be considered tentative pending confirmation with a larger sample. Another study by Morales et al. (2008) focused on single stars of spectral type late K and M, and used the strength of the H α emission to distinguish active from inactive objects. They found that single active low-mass stars show similar radius and temperature anomalies as the stars in well-studied eclipsing binaries, implying that the problem is independent of whether the object is in a binary or not. More recently Stassun et al. (2012) also made use of H α measurements for single stars, as well as X-ray activity measurements in binaries, and derived empirical relations between the radius inflation and temperature suppression as a function of activity.

On the theoretical side, Mullan & MacDonald (2001) compared active and inactive single stars as measured by their X-ray luminosity, and were able to explain their systematically different global properties (temperatures and radii) both qualitatively and quantitatively using custom models incorporating magnetic fields. They parametrized them in terms of a magnetic inhibition parameter δ (the ratio of the magnetic to total energy density) to describe the reduction in the efficiency of convection in the stellar interiors. They recently applied the same scheme to successfully model the components of CM Dra (MacDonald & Mullan 2012). Earlier studies such as that by D'Antona, Ventura & Mazzitelli (2000) had also shown how magnetic fields change the global structure of stars and how models that incorporate these effects can be made to match the observations. More recently Chabrier et al. (2007) were also able to explain the radius and temperature anomalies in lowmass stars with a different theoretical approach, and suggested that two different manifestations of stellar activity can change the sizes and temperatures of low-mass stars: magnetic inhibition, and spot coverage. They modeled both and showed that with a reduced mixing length parameter $\alpha_{\rm ML}$ and a suitable spot-filling factor β (fraction of the star covered by spots) it is possible to match the inflated radii and suppressed temperatures of these objects.

According to the above theoretical studies cool spots have a real and detectable effect on the global stellar properties because they reduce the effective radiating area, puffing up the star and reducing its temperature. But they can also have a deleterious effect on the measurements, specifically on the radius determinations. Spots typically cause modulations and/or features in the light curves, affecting the shape of the eclipses. Numerical simulations carried out by Morales et al. (2010) showed that non-negligible biases in the radii can result, depending on the spot distribution. They found that polar spots (the kind often expected in rapidly rotating stars, and actually seen in Doppler tomography) have the largest effect, causing the radius to be overestimated by up to 3-6%. Note that this bias goes in the same direction as the observed discrepancies between models and observations, and would thus tend to alleviate the differences if we were able to avoid it. The effect on the mass determinations, on the other hand, was found to be less serious (0.5-1%). For spot distributions less concentrated to the poles the effects on the radius are smaller and more random in nature.

These systematic errors are especially worrisome when considering that spots tend to change or to come and go on active stars. A recent illustration of this was given by Windmiller, Orosz & Etzel (2010), who re-observed the wellstudied system GU Boo (López-Morales & Ribas 2005). They reanalyzed the original light curves, as well as two new ones they obtained two years later, and found differences in the measured radii at the level of 2%. They attributed most of this to the spots (with perhaps some contribution from the modeling technique), which had changed visibly in the intervening years.

The lesson from the empirical and numerical evidence described above is that spots can cause systematic measurement errors in the radii at the level of several percent, if the stars are sufficiently active. This certainly complicates the picture, and suggests the need to investigate these effects carefully for each new system. At the very least, it may argue for being more conservative in stating one's measurement uncertainties for low-mass stars.

7 Final remarks

The last few years have seen excellent progress in measuring fundamental properties of low-mass stars, and in understanding the underlying causes of the radius and temperature discrepancies with standard (non-magnetic) models. Recent models that incorporate the effects of magnetic fields are able to match the measured properties of low-mass stars, at the expense of adding one or two more free parameters. It remains to be seen if these or other equivalent parameters can be put in terms of some easily measurable quantity, which would then allow more stringent tests of theory.

With transiting exoplanet and other photometric surveys in full swing, prospects are good that the empirical evidence will continue to build. The most useful studies will be those using complete and high-quality data analyzed with appropriate techniques, and that pay careful attention to systematic errors, especially those related to spots. In additional to measuring masses and radii in eclipsing binaries, observers should endeavor to determine also the effective temperatures and metallicity whenever possible, and to estimate the activity levels, which clearly play an important role.

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