

A nonextensive approach to the stellar rotational evolution

I. F and G type stars

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ABSTRACT

The pioneering study by Skumanich (1972) showed that the rotational velocity of G-type Main-Sequence (MS) stars decreases with stellar age according to $\langle v \sin i \rangle \propto t^{-1/2}$. This relationship is consistent with simple theories of angular momentum loss from rotating stars, where an ionized wind is coupled to the star by a magnetic field. The present study introduces a new approach to the study of stellar rotational braking in unsaturated F and G type stars limited in age and mass, connecting angular momentum loss by magnetic stellar wind with Tsallis nonextensive statistical mechanics. As a result, we show that the rotation-age relationship can be well reproduced using a nonextensive approach from Tsallis nonextensive models. Here, the index q , which is related to the degree of nonextensivity, can be associated to the dynamo process and to magnetic field geometry, offering relevant information on the level of stellar magnetic braking for F- and G-type Main-Sequence stars.

Key words: Stars: evolution – Stars: rotation – Stars: statistics.

1 INTRODUCTION

The relationship between stellar rotation and age is an exciting topic in astrophysics posing, a number of unanswered questions, including: 1) How is the stellar rotation-age relationship dependent on mass? 2) How does rotation influence the stellar dynamo, as well as chromospheric and coronal heating mechanisms? 3) How do initial star formation systems affect rotational evolution? 4) How do stellar angular momentum and stellar activity evolve over time?

Skumanich (1972)’s pioneering study showed that $v \sin i$ of G-type Main-Sequence (MS) stars for Hyades and Pleiades measured by Kraft (1967) were consistent with $\langle v \sin i \rangle \propto t^{-1/2}$, where t is the stellar age, v is the equatorial rotational velocity of the star and i the inclination angle of the rotational axis to the line of sight. This relationship is in line with simple theories of angular momentum loss from rotating stars, where an ionized wind is coupled to the star by a magnetic field (Schatzman 1962). Indeed, angular momentum loss due to stellar winds is generally believed to be responsible for the Skumanich relationship, but the exact dependence of rotation on age has not been fully established (Kawaler 1988; Krishnamurthi *et al.* 1997). Barry *et al.* (1987) and Soderblom *et al.* (1991) reported similar qualitative results for solar-type stars, but with power-law exponents ranging from $-1/2$ (corresponding to the Skumanich relation) to $-4/3$. Pace & Pasquini (2004) claimed

that these power-laws do not fit the age-activity-rotation of G dwarf stars in open clusters. According to these authors, a $t^{-5/2}$ law is more consistent with the observations. Despite the differences in rotation-activity-age relationships obtained to date, most of the data strongly suggest that this relationship is deterministic and not merely a statistical artifact. Indeed, rotation data for stars at different evolutionary stages and environments, in particular for stars in open clusters, have confirmed the deterministic nature (e.g.: Barnes & Kim 2010; Reiners and Mohanty 2012) Barnes (2003), Mamajek & Hillenbrand (2008), James *et al.* (2010) and Meibom *et al.* (2011a,b).

In this study we present a new approach to the study of stellar rotational braking in F and G type stars, connecting angular momentum loss by magnetic stellar wind (Kawaler 1988; Chaboyer *et al.* 1995) with Tsallis’ nonextensive statistical mechanics (Tsallis 1988). In Section 2, we revisit parametric models for angular momentum loss by magnetic stellar wind, with an emphasis on a modified Kawaler model. Section 3 describes our nonextensive approach, in Section 4 we describe the observational data, and Section 5 compares our model with $v \sin i$ measurements for a large sample of F- and G-type single main-sequence stars. Finally, conclusions are presented in Section 5.

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2 REVISITING ANGULAR MOMENTUM LOSS BY MAGNETIC STELLAR WIND

Following the theory of angular momentum loss elaborated by Mestel (1968, 1984) and Mestel & Spruit (1987), Kawaler (1988) developed a general parametrization showing that the rate of angular momentum loss is proportional to $\Omega^{1+4aN/3}$, where Ω is angular velocity, a denotes the dynamo relationship and N is a measurement of magnetic field geometry. The problem with angular momentum loss due to magnetic stellar wind is its dependence on these two parameters, a and N . Kawaler (1988) simplified the dynamo relationship by assuming that the total magnetic field is proportional to the rotation rate to some power, denoted by $B_0 \propto \Omega^a$, where a varies between 1 and 2 for the unsaturated domain and 0 for the saturated domain. As demonstrated by Chaboyer *et al.* (1995), the N parameter exhibits a wide range, varying from 0 to 2, where 0 represents dipole field geometry and 2 a radial field. Moreover, these authors have shown that angular momentum loss from high rotation stars cannot be described by Kawaler's relationship. The same authors and subsequent studies (Reiners and Mohanty 2012) have also demonstrated that magnetic fields at high rotation rates must saturate, following the form $dJ/dt \propto \Omega$. In addition, studies by Matt and Pudritz (Matt & Pudritz 2005, 2008a,b) have explored the role of stellar winds to the spin-up and spin-down torques expected to arise from the magnetic interaction between a slowly rotating pre-main-sequence star and its accretion disk, demonstrating that if stellar mass outflow rates are substantially greater than the solar rate, the stellar winds can carry off orders of magnitude with more angular momentum than can be transferred to the disk.

In terms of parametrization, there are two parameterized forms known as modified Kawaler's relationships:

$$\frac{dJ}{dt} = -K_w \Omega \omega_{sat}^{4aN/3} \left(\frac{R}{R_\odot} \right)^{2-N} \cdot \left(\frac{M}{M_\odot} \right)^{-N/3} \left(\frac{\dot{M}}{10^{-14}} \right)^{1-2N/3} \quad (1)$$

for $\Omega \geq \omega_{sat}$ and

$$\frac{dJ}{dt} = -K_w \Omega^{1+4aN/3} \left(\frac{R}{R_\odot} \right)^{2-N} \cdot \left(\frac{M}{M_\odot} \right)^{-N/3} \left(\frac{\dot{M}}{10^{-14}} \right)^{1-2N/3} \quad (2)$$

for $\Omega < \omega_{sat}$.

Here, ω_{sat} is the threshold angular velocity beyond which saturation occurs, K_w is a calibration constant and R , M and \dot{M} represent the stellar radius, mass and mass-loss rate in units of $10^{-14} M_\odot \text{yr}^{-1}$, respectively. In this regard, ω_{sat} completes the triplet-order parameter. In fact, this saturation threshold is an important ingredient for theoretical models of stellar angular momentum evolution. For instance, Krishnamurthi *et al.* (1997) and Sills, Pinsonneault and Terndrup (2000) suggest that the saturation threshold depends on stellar mass and is inversely proportional to the global convective overturn timescale, which is related to the Rossby number, an indicator of the strength of dy-

namo magnetic activity. By contrast, Reiners and Mohanty (2012) argue that ω_{sat} does not depend on mass. According to Pizzolato *et al.* (2003), the saturation phenomenon is also based on magnetic proxies and its level can be inferred, irrespective of angular momentum evolution arguments, which, do not require a Rossby number for either regime. These authors also argue that the X-ray emission level from saturated stars depends only on bolometric luminosity, whereas for non-saturated stars the emission level depends only on the rotational period. This implies a direct mass-dependent ω_{sat} (Krishnamurthi *et al.* 1997). Since this timescale does not change much in our model, we neglect its effect for a given mass (Barnes and Sofia 1996; Krishnamurthi *et al.* 1997; Pizzolato *et al.* 2003).

In the present study we assume that since the moment of inertia I and stellar radius R changes slowly during MS evolution, the angular momentum loss law can be simplified by considering the limit of dJ/dt for constants I and R , that is, this loss law is fully specified by the rotational velocity, while the star is braked by the stellar wind (Bouvier, Forestini & Allain 1997). We also consider that in the absence of angular momentum loss, equatorial rotational velocity v can be determined by the simple condition of angular momentum conservation, denoted by

$$J = \left(\frac{I}{R} \right)_{const.} v. \quad (3)$$

It is important to underline that in our model we consider a solid-body rotation, although this assumption can directly affect spin down effects for F-type stars. For a more complete model, it is necessary to take in consideration other physical aspects such as core-envelope decoupling (Cameron & Li Jianke 1993; Allain 1998), as well as the evolutionary behavior of the moment of inertia (Wolff & Simon 1997; Zorec & Royer 2012).

Two pairs of equations were combined to obtain the time dependence of v . First, from eqs. (1) and (3) we have

$$v(t) = v_0 \exp \left[-\frac{(t-t_0)}{\tau_1} \right], \quad (t_0 \leq t < t_{sat}) \quad (4)$$

with

$$\tau_1 \equiv \left[f(\Lambda) \omega_{sat}^{4aN/3} \right]^{-1} \quad (5)$$

for the saturated domain, where $f(\Lambda)$ is denoted by

$$f(\Lambda) = \frac{K_w}{I} \left(\frac{R}{R_\odot} \right)^{2-N} \left(\frac{M}{M_\odot} \right)^{-N/3} \left(\frac{\dot{M}}{10^{-14}} \right)^{1-2N/3} \quad (6)$$

where I denotes the moment of inertia of the star and Λ denotes a parameter set given by $\Lambda = \{R, M, \dot{M}, N\}$.

On the other hand, by combining eqs. (2) and (3) we obtain the equation

$$v(t) = v_{sat} \left[1 + \frac{(t-t_{sat})}{\tau} \right]^{-3/4aN}, \quad (t \geq t_{sat}) \quad (7)$$

with

$$\tau \equiv \left[\frac{4aN}{3} f(\Lambda) \omega_{sat}^{4aN/3} \right]^{-1}. \quad (8)$$

In this respect, the τ_1 and τ are the MS spin-down timescales in the saturated and unsaturated regimes, respectively. The other parameters, t_0 , v_0 and t_{sat} , are the age at

Table 1. Best parameter values of our unsaturated model using eq. (14). The values of N using eq. (15) also are shown.

Stars	$\langle M/M_{\odot} \rangle$	q	$N_{a=2} - N_{a=1}$
F0-F5	1.36	1.80 ± 0.1	0.30 – 0.60
F6-F9	1.22	1.96 ± 0.2	0.36 – 0.72
G0-G5	1.11	2.18 ± 0.2	0.44 – 0.89
G6-G9	0.98	4.38 ± 0.6	1.27 – 2.53
All F		1.91 ± 0.1	0.34 – 0.68
All G		2.18 ± 0.4	0.44 – 0.89

which the star arrives on the MS, the rotational velocity at that time and subsequent age at which the star slows down to values below the critical velocity v_{sat} , respectively (Reiners and Mohanty 2012). Time t_{sat} can be determined by setting $v(t) = v_{sat}$ in eq. (4). Thus, we have

$$t_{sat} = t_0 + \tau_1 \ln \left(\frac{v_0}{v_{sat}} \right). \quad (9)$$

Eq. (4) reveals that in the saturated regime the star slows down exponentially from time t_0 up to velocity v_{sat} in time t_{sat} . Eq. (7) shows that from this time onward, the spin-down rate decreases to a power law given by $-3/4aN$. Thus, the star remains within a factor of a few v_{sat} for the remainder of its MS lifetime.

3 NONEXTENSIVE APPROACH TO THE STELLAR ROTATION-AGE RELATION

This section details a nonextensive approach for the modified Kawaler model divided into two classes: saturated and β -saturated models. Both have the same unsaturated model, but with different saturated timescale t_{sat} .

In general, for the unsaturated regime, it can be considered that the rotation-age connection follows a Zipf-like power law (Zipf 1949, 1965), described as

$$\frac{v(t)}{v_0} = \left(1 + \frac{t}{\tau} \right)^{\alpha}, \quad (10)$$

where τ is the timescale that controls the crossover between the initial plateau and the power-law domain, characterized by the exponent α , and v_0 is a normalization constant. Equation (10) was deduced within Tsallis' nonextensive theory (Tsallis, Bemsiki & Mendes 1999; Lyra *et al.* 2003), exhibiting fractal properties and long-range memory. This theory demonstrates that the usual exponential for saturated fields can be recovered if $\alpha \rightarrow \infty$.

3.1 Saturated and unsaturated regimes

Applying Tsallis statistics (Tsallis 2004), we can assume that eq. (4) follows a simple linear differential equation in the form

$$\frac{d}{dt} \left(\frac{v}{v_0} \right) = -\lambda_1 \left(\frac{v}{v_0} \right), \quad (11)$$

from which

$$v(t) = v_0 \exp[-\lambda_1(t - t_0)], \quad (12)$$

where λ_1 denotes the classical Lyapunov coefficient.

According to Lyra *et al.* (2003), in contrast to the exponential behavior presented in eq. (12), eq. (11) must be replaced with a slower power law at critical points where long correlations develop. In this context, a similar expression can be created to characterize the possible behavior of stellar rotation distribution in an unsaturated regime, denoted by

$$\frac{d}{dt} \left(\frac{v}{v_{sat}} \right) = -\lambda_q \left(\frac{v}{v_{sat}} \right)^q \quad (\lambda_q \geq 0; q \geq 1). \quad (13)$$

Integrating the equation above, we have

$$v(t) = v_{sat} [1 + (q - 1)\lambda_q(t - t_{sat})]^{1/(1-q)}, \quad (14)$$

where $\exp_q(x) \equiv [1 + (1 - q)x]^{1/(1-q)}$ denoted by q -exponential¹ and λ_q the nonextensive Lyapunov coefficient.

Thus, in the nonextensive scenario, eq. (7) is well described as a non-linear differential equation in form (13) with solution (14). When $q = 1$, eq. (12) is recovered, indicating that a system governed by a saturated regime is in thermodynamic equilibrium. By contrast, when q differs from unity, a system controlled by unsaturation is out of equilibrium.

In agreement with our model, for the unsaturated regime, exponent q can be described by a pair (a, N) given by

$$q \equiv 1 + \frac{4aN}{3}, \quad (15)$$

whereas for saturated regime $q = 1$ due to the usual exponential function. From above equations, we can estimate a relationship between the characteristic time τ_1 and τ_q given by

$$\frac{\tau_1}{\tau_q} = (q - 1) \frac{\lambda_q}{\lambda_1}, \quad (16)$$

in this context, λ_s denote the braking strength.

The q -parameter is related to the degree of nonextensivity that emerges within thermodynamic formalism proposed in Tsallis (1988). As revealed in eq. (15), q is a function of magnetic field topology N and dynamo law a , which depend on stellar evolution. According to Kawaler (1988), small N values result in a weak wind that acts on the MS phase of evolution, while winds with large N values remove significant amounts of angular momentum early in the Pre-Main-Sequence (PMS). In this phase, for a given mass, maximum rotational velocity v decreases as N increases. This result is significant because, within a thermostistical framework in which eq. (13) naturally emerges, for a given value (a, N) we obtain the scale laws found in the literature, such as those proposed by Skumanich (1972) and Soderblom *et al.* (1991).

3.2 β -Saturated regime

As reported by Ivanova and Taam (2003), high rotation rates can differ significantly from the exponential decay, without which the angular momentum loss rate necessarily saturates the magnetic field (Chaboyer *et al.* 1995). As such, we used a more general relationship than that described by eq. (1).

¹ See Borges (2004) for a review of q -calculus.

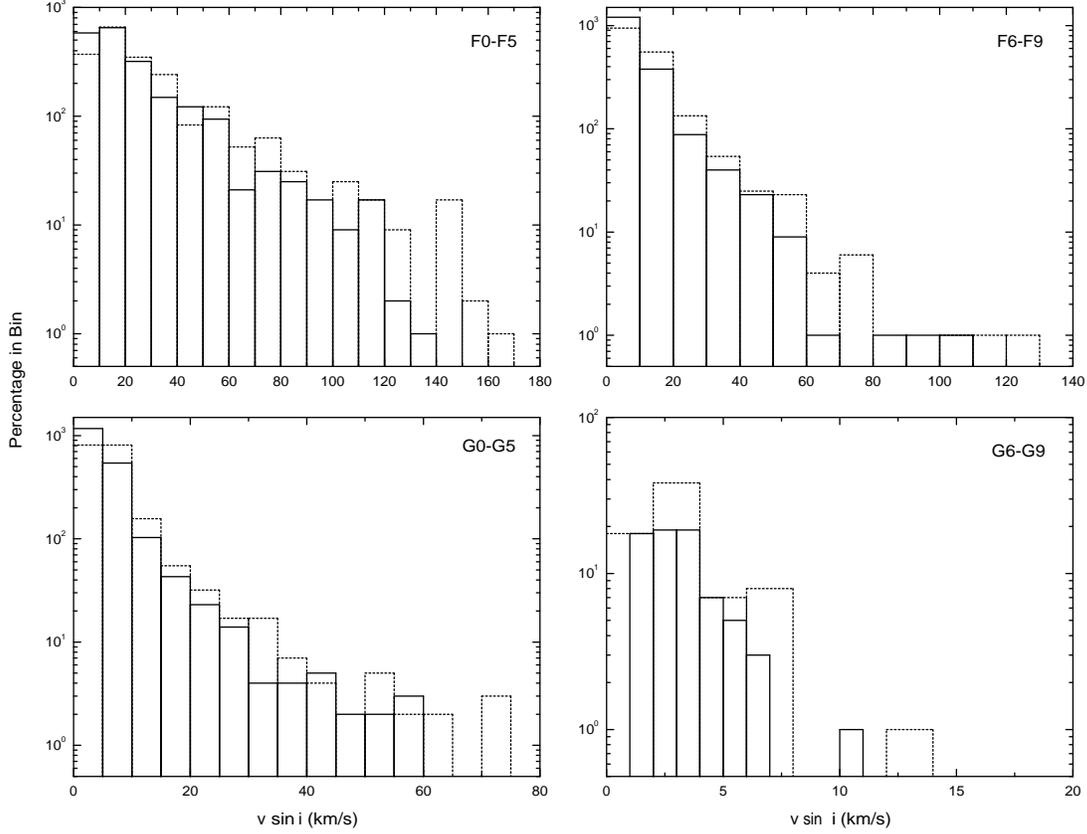


Figure 1. Semi-log plot of the rotational velocity distribution for the sample of F and G-type stars. Solid lines are referred to $v \sin i$ data, while short dashed lines are the data multiplied by the statistical correction factor $4/\pi$.

Thus, this equation can be rewritten considering that the dJ/dt rate depends on the Ω , as follows:

$$\frac{dJ}{dt} = -K_w \Omega^\beta \omega_{sat}^{1+4aN/3-\beta} \left(\frac{R}{R_\odot} \right)^{2-N} \cdot \left(\frac{M}{M_\odot} \right)^{-N/3} \left(\frac{\dot{M}}{10^{-14}} \right)^{1-2N/3}, \quad (t_0 \leq t < t_{sat}), \quad (17)$$

for $\beta = 1$, we have the saturated domain, while for $\beta > 1$ can be describe as a *quasisaturated* regime, defined here as β -saturated. Deriving eq. (3) and comparing with eq. (17), we obtain that

$$v(t) = v_0 \left[1 + \frac{(t - t_0)}{\tau_\beta} \right]^{\frac{1}{1-\beta}} \quad (18)$$

with

$$\tau_\beta \equiv \left[(\beta - 1) f(\Lambda) \left(\frac{v_0}{v_{sat}} \right)^{\beta-1} \omega_{sat}^{4aN/3} \right]^{-1}. \quad (19)$$

Following the same procedure used to obtain $v(t)$ in the previous cases, we have

$$v(t) = v_0 [1 + (\beta - 1) \lambda_\beta (t - t_0)]^{\frac{1}{1-\beta}}, \quad (20)$$

where, β is defined in interval $1 \leq \beta < 1 + 4aN/3$.

In this regime, a generalized expression to (9) can be obtained using the definition of so-called q -logarithm: $\ln_q(x) = \frac{x^{1-q} - 1}{1-q}$. Thus,

$$t_{sat} = t_0 + (\beta - 1) \tau_\beta \ln_\beta \left(\frac{v_0}{v_{sat}} \right), \quad (21)$$

where for $\beta = 1$ the standard logarithm is recovered in (9).

In this general case, we can extrated the rate between the time τ_q and τ_β , where we obtain

$$\frac{\tau_\beta}{\tau_q} = \left(\frac{q-1}{\beta-1} \right) \frac{\lambda_q}{\lambda_\beta} \quad (22)$$

where τ_1 and τ_β are related by

$$\frac{\tau_\beta}{\tau_1} = \left(\frac{1}{\beta-1} \right) \left(\frac{v_0}{v_{sat}} \right)^{1-\beta} \quad (23)$$

if $\beta = 1$, we recovered the rate between τ_1 and τ_q in case of the unsaturated-saturated model.

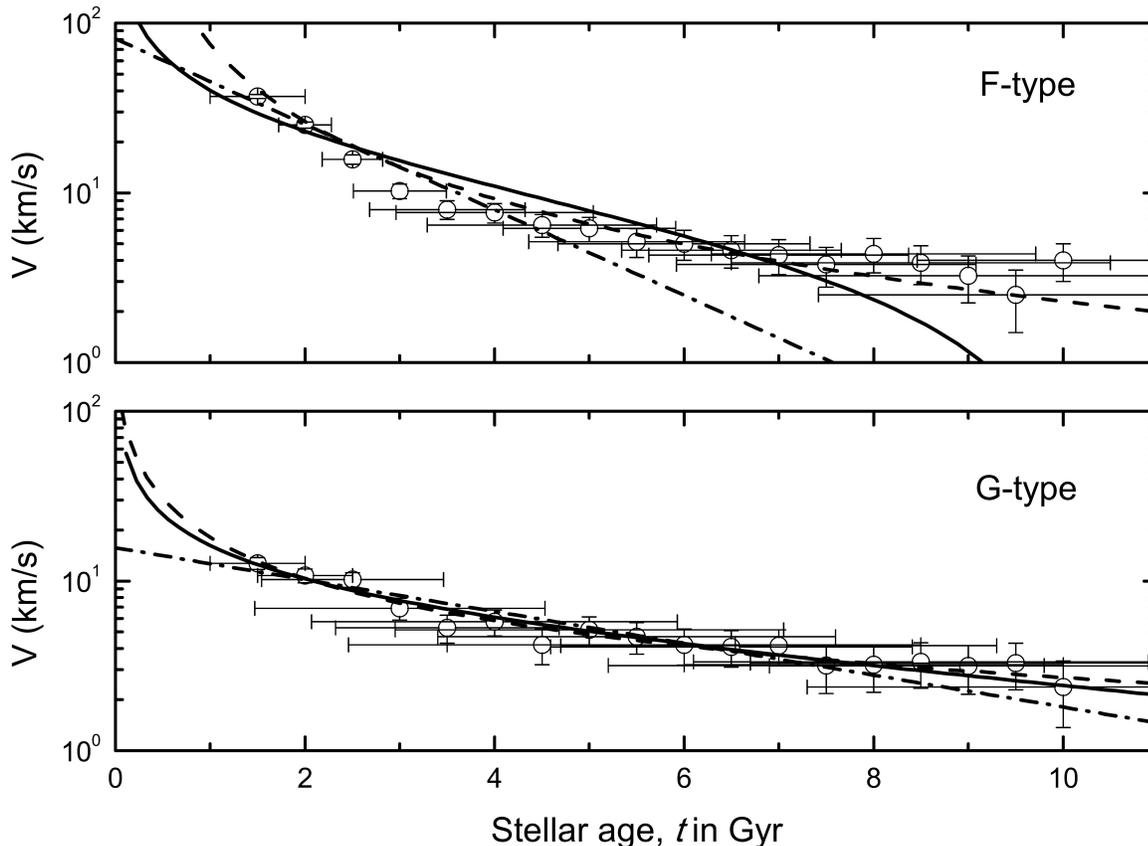


Figure 2. Semi-log plot of the stellar rotation-age relationship for the sample of F and G-type stars used in the present study. Upper and lower panels display mean projected rotational velocity, $\langle v \sin i \rangle$, by age intervals of 0.5 Gyr for F and G-type stars, respectively, represented by the solid circles. In both panels, continuous lines represent the fit using Skumanich’s law, and dash-dotted and dashed lines correspond to the fits using eqs. (12) and (14), respectively. Error bars for $v \sin i$ and age are also indicated.

4 THE STELLAR SAMPLE AND OBSERVATIONAL DATA

Our study is based on the kinematically unbiased and magnitude-limited sample of nearby F and G dwarf stars from the Geneva–Copenhagen Survey (CGS) of the Solar neighbourhood, volume complete to ~ 40 pc, carried out by Nordström *et al.* (2004) and rediscussed by Holmberg, Nordström & Andersen (2007), which contain age, metallicity, mass, projected rotational velocity $v \sin i$ and kinematic properties for about 14000 F and G dwarfs. Readers are referred to these authors for observational procedure and data reduction, but let us briefly discuss the quality of projected rotational velocity $v \sin i$, age and mass measurements, given that these parameters are the most sensitive in the present study.

For the vast majority of stars (12941 stars) the $v \sin i$ measurements were computed from observations carried out with the CORAVEL spectrometers (Baranne *et al.* 1979; Mayor 1985), applying (Benz & Mayor 1980, 1984)’s calibrations. Such a procedure gives $v \sin i$ values with a precision

of 1 km s^{-1} , at least for rotations lower than about 30 km s^{-1} (De Medeiros & Mayor 1999).

A total of 833 stars rotation measurements were determined from observations carried out with the digital spectrometer (Latham 1985) at the Harvard–Smithsonian Center for Astrophysics, CfA, in particular for stars rotating too rapidly for CORAVEL. For these stars, the $v \sin i$ values were computed on the basis of the best-fitting template spectrum (Nordström *et al.* 1994, 1997). As shown by these authors, for slowly-rotating stars, typically for $v \sin i$ values lower than 20 km s^{-1} , there is a very good agreement between data based on CfA and CORAVEL observations. As the vast majority of the present stellar working sample have rotation below 20 km s^{-1} , we can consider that data on rotation used in the present analysis have the same quality, with a precision of about 1 km s^{-1} .

We assumed that the mean value for a random distribution of inclinations $\sin i$ is given by $4/\pi$ (Chandrasekhar & Munch 1950). As mentioned by Reiners and Mohanty (2012), this correction is only necessary in a statistically averaged sense and not for comparison of single observed pop-

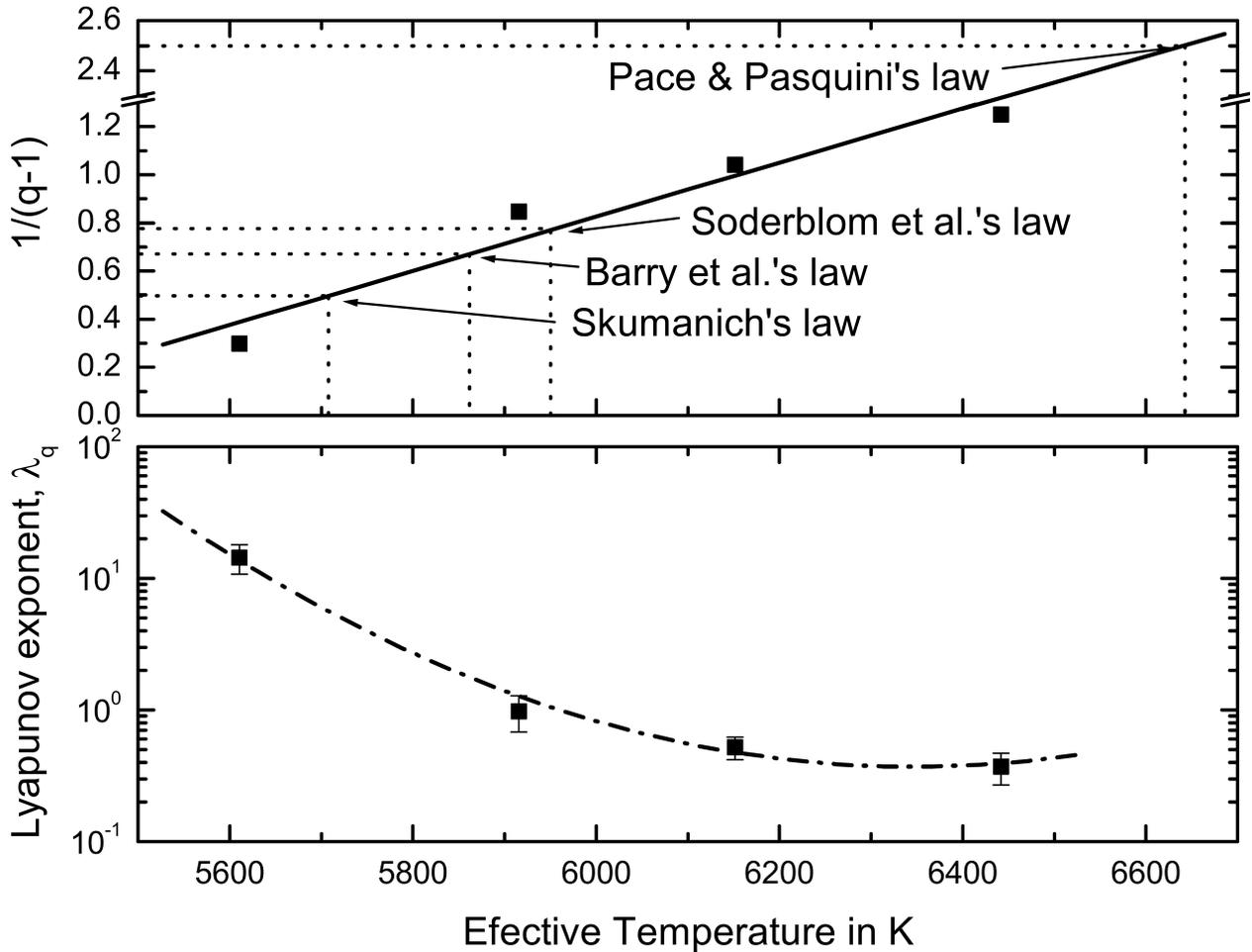


Figure 3. The Effective Temperature dependences of q (upper panel), λ_q (lower panel) used to fit the our sample segregated by different interval of spectral type. Upper figure indicates a linear dependende between the effective temperature and parameter $1/(q - 1)$. For instance, Schumanich law ($1/(q - 1) = 0.5$) corresponds to stars with solar effective temperature. While that Pace & Pasquini law corresponds to hottest stars.

ulations. Such a fact is valid for the present stellar sample, which is unbiased and magnitude-complete. In this sense statistical variations in the $\sin i$ distribution should not significantly change the presence or location of the very steep transition from a large population of undetected $v \sin i$ to a similarly large population of high $v \sin i$ observed (Reiners and Mohanty 2012). Fig. 1 shows the projected velocity distribution per bin uncorrected and corrected by factor $4/\pi$, in which no significant $\sin i$ effects can be observed.

Good age measurements are, of course, crucial when investigating the rotation-age relationship. Most of the previous studies on this subject have used ages estimated from chromospheric activity. Nevertheless, such a procedure is not valid for stars at the age of the Sun, because chromospheric emission essentially vanishes at this stage. The theoretical isochrone age is then the best choice, although important discrepancies subsist, in particular for old low-mass stars. The ages given in the GCS were then obtained on the basis of the stellar isochrone procedure Nordström *et al.* (2004), using the Bayesian computational technique of Jørgensen and Lindegren (2005). A revision of the data was carried

out by Holmberg, Nordström & Andersen (2007), taking into account new temperature and metallicity calibrations. According Nordström *et al.* (2004) and Holmberg, Nordström & Andersen (2007) for 81% of the presumably single stars in the original sample of the CGS, age estimations based, on their procedure, have errors below 50%.

Nordström *et al.* (2004) have also estimated stellar masses using theoretical isochrone analysis, with individual errors averaging about $0.05 M_{\odot}$, and metallicities from Strömgren *ubvyB* photometry. As shown by these authors, the distribution of their estimated metallicities obeys a Gaussian distribution with a mean of -0.14 and dispersion 0.19 dex. The distribution of the metallicities from Holmberg, Nordström & Andersen (2007) for the 5835 single stars used in the present analysis shows a mean value $\langle [Fe/H] \rangle = -0.18$ and dispersion of 0.21 dex. Clearly, one observes that our working sample is mostly composed of solar-type stars, in spite of the presence of a low percentage of stars with slightly sub-solar metallicities. Recently, Casagrande *et al.* (2011) re-analyzed most of the parameters for the CGS via the infrared flux method (IRFM). Accord-

ing to these authors, this method, in comparison to previous CGS calibrations, reveals differences in measurement of effective temperature leading to a much better agreement between stars and isochrones in the HR diagram.

In order to test our nonextensive approach for stellar rotational magnetic braking, we used $v \sin i$ and ages from the referred Geneva-Copenhagen survey for F- and G-type single stars (Nordström *et al.* 2004; Holmberg, Nordström & Andersen 2007), with ages limited to 10 Gyr and masses of $0.90M_{\odot} \leq M \leq 2.0M_{\odot}$. This upper limit for ages was defined by the star's lifetime on the MS and the age of the Galactic disk (Epstein & Pinsonneault 2012), while the upper limit for masses (or spectral type) is based on the different internal structure of the stars in our sample. These age and mass limits were chosen to avoid the presence of stars exhibiting the most significant uncertainties for these parameters, namely the oldest and lowest mass stars, as well as the highest mass stars. Based on these criteria, we obtained a final working sample of 5780 single stars consisting of 3790 F- (2050 with spectral type between F0 and F5 and 1750 between F6 and F9) and 1990 G- (1919 with spectral type between G0 and G5 and 73 between G6 and G9) type stars.

5 TESTING THE MODEL WITH UNSATURATED STARS

First of all, stars in the present working sample can be considered to be in the unsaturated regime, indicating that the computation of the values of the parameter β is not necessary. Indeed, all the stars are aged more than 1 Gyr, as described in Section 4.

Different studies in the literature (*e.g.*: (Krishnamurthi *et al.* 1997; Bouvier, Forestini & Allain 1997; Epstein & Pinsonneault 2012)) use the rotation distribution of young open clusters to estimate initial conditions. Here, we follow the same approach, by considering the rotation distribution of T Tauri stars. In agreement with initial conditions, we compute the best-fit $[q, \lambda_q]$ doublet. Fig. 2 and 3 shows the results of the present analysis, where the mean of $v \sin i$ by age intervals of 0.5 Gyr are shown for the aforementioned sample of F and G-type stars. In Fig. 2, the upper and lower panels also display the best fits applying the Skumanich law, as well as eq. (14). The best-fit parameters for the rotation-age relationship using this equation are depicted in Fig. 3.

Fig. 2 clearly shows that the saturated model does not fit the data for either type of star aged > 1 Gyr. This suggests that the stars in our sample are still under the effect of magnetic torque. Additionally, there is a positive gradient for index q from the hottest to the coolest stars, namely from F- to G-type stars, suggesting that at a given age the coolest stars have a stronger rotational decline, reaching the slow rotation regime faster than the hottest stars. Moreover, there is also a positive gradient for braking strength λ_q from the hottest to the coolest stars. Indeed, from Table 1 demonstrates that the present model clearly depend on stellar mass.

According to q values presented in Fig. 3, magnetic field geometry is essentially (quasi)dipolar, irrespective of the value of a . For this type of topology, our results indicate that for F early-type stars the Pace and Pasquini law

($q = 1.4$) is the most appropriate, while for late-type stars the Soderblom *et al.*'s ($q = 2.3$) law can be considered a good approach. In case of G stars the Skumanich ($q = 3$) law is the most appropriate for early-type and the Soderblom *et al.* law is for late-type.

6 CONCLUSIONS

In summary, our study presents a new statistical approach for stellar rotational braking. The present analysis shows that the rotation-age relationship can be well reproduced using a nonextensive approach from statistical mechanics, namely the Tsallis nonextensive models. According to different studies (*e.g.*: Rutten & Pylyser (1988); Matt *et al.* (2012)), a theoretical model for magnetic braking should consider several parameters, (*e.g.* magnetic field geometry, mass loss rate and coronal temperature), which to data are not represented in the different laws describing the rotation-age relations. The nonextensive analysis proposed here considers some of these parameters, and offers the possibility of studying the stellar rotational braking behaviour of different classes of stars, based on the same approach.

Our theoretical-observational results reveal that each spectral type range is characterized by a specific entropy production, this suggests the occurrence of different rotational velocity probability distribution with high energy tails as proposed by de Freitas & De Medeiros (2012). Moreover, these results suggest that increasing index q behaviour between the saturated and unsaturated regimes as a function of spectral type may be due to a phase transition magnetic process that occurs in the stellar dynamo dynamics. According to Bouvier, Forestini & Allain (1997), low mass stars retain the memory of their initial angular momentum up to an age of about 1 Gyr, as opposed to solar type stars, where the memory is retained up to few 10^8 years. This memory can be quantified using the entropic index q . The fact that this index increases from F to G stars, indicates that long-range memory grows at the same scale. Higher mass stars do not maintain this type of memory for long time, because the interaction of the young star with its circumstellar disk is much shorter than their very low-mass counterparts, which retain constant angular velocity.

Finally, a comparison with observational data, namely projected rotational velocity $v \sin i$, shows that our nonextensive approach closely fits the decrease in stellar rotation with age for F- and G-type main-sequence stars. Indeed, the behavior of the index q , which is related to the degree of nonextensivity, and in the present approach can be described by a and N , associated to the dynamo process and magnetic field geometry, respectively offers relevant information concerning the level of magnetic braking. At a given age the coolest stars have a stronger rotational decline, reaching the slow rotation regime faster than the hottest stars.

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