Describing galaxy weak lensing measurements from tenths to tens of Mpc and up to $z \sim 0.6$ with a single model

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ABSTRACT

The clustering of galaxies and the matter distribution around them can be described using the halo model complemented with a realistic description of the way galaxies populate dark matter haloes. This has been used successfully to describe statistical properties of samples of galaxies at z < 0.2. Without adjusting any model parameters, we compare the predicted weak lensing signal induced by Luminous Red Galaxies to measurements from SDSS DR7 on much larger scales (up to ~ $90 h_{70}^{-1}$ Mpc) and at higher redshift ($z \sim 0.4$). We find excellent agreement, suggesting that the model captures the main properties of the galaxy-dark matter connection. To extend the comparison to lenses at even higher redshifts we complement the SDSS data with shape measurements from the deeper RCS2, resulting in precise lensing measurements for lenses up to $z \sim 0.6$. These measurements are also well described using the same model. Considering solely these weak lensing measurements, we robustly assess that, up to $z \sim 0.6$, the number of central galaxies as a function of halo mass is well described by a log-normal distribution with scatter $\sigma_{\log L_c} = 0.146 \pm 0.011$, in agreement with previous independent studies at lower redshift. Our results demonstrate the value of complementing the information about the properties of the (lens) galaxies provided by SDSS with deeper, high-quality imaging data.

Key words: galaxies: halos — large-scale structure of Universe — dark matter — gravitational lensing — methods: statistical

1 INTRODUCTION

Since the advent of large and homogeneous galaxy surveys, it has become possible to constrain the relation between the observed properties of galaxies and their host dark matter haloes with ever increasing precision, albeit in a statistical sense. In particular, studies of the observed abundances and clustering properties of galaxies (e.g. Vale & Ostriker 2004; Conroy et al. 2006; Shankar et al. 2006; Vale & Ostriker 2006; Yang et al. 2008; Moster et al. 2010; Guo et al. 2010; Behroozi et al. 2010; Moster et al. 2012; Guzzo et al. 2000; Norberg et al. 2001, 2002; Zehavi et al. 2005; Wang et al. 2007) have played a crucial role in establishing this relation with increasing detail.

Complementing these methods, the weak gravitational lensing signal around galaxies of different observed properties (galaxy-galaxy lensing) has emerged as another powerful technique to constrain this relation. Since the first detections (e.g. Brainerd et al. 1996;

Griffiths et al. 1996; Hudson et al. 1998), the galaxygalaxy lensing signal is now detected routinely as a function of the properties of the lens galaxies, thanks to multi-wavelength data becoming readily available (e.g. Fischer et al. 2000; McKay et al. 2001; Hoekstra et al. 2003, 2005; Sheldon et al. 2004; Mandelbaum et al. 2006; Heymans et al. 2006; Parker et al. 2007; Mandelbaum et al. 2008; van Uitert et al. 2011; Choi et al. 2012). The systematics involved in these measurements have also been studied in great detail (see e.g. Mandelbaum et al. 2005, 2006, 2008). The main application remains the study of the galaxy-dark matter connection, such as measurements of scaling relations between halo mass and baryonic prop-(e.g. Hoekstra et al. 2005; Mandelbaum et al. erties 2006;Cacciato et al. 2009; Leauthaud et al. 2010;van Uitert et al. 2011; Choi et al. 2012), constraints on the halo properties (e.g. Hoekstra et al. 2004;Mandelbaum et al. 2006;Limousin et al. 2007;Mandelbaum et al. 2008; van Uitert et al. 2012), and measurements of bias parameters (e.g. Hoekstra et al. 2001, 2002; Sheldon et al. 2004). More recently galaxy-galaxy

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lensing has also been used as a cosmological probe in combination with galaxy abundance and/or clustering measurements (More et al. 2013; Cacciato et al. 2013; Rozo et al. 2010; Zu et al. 2012; Mandelbaum et al. 2012).

There is growing scientific interest in probing the cosmic evolution of structure formation in the Universe, which is now becoming possible thanks to new and forthcoming galaxy surveys. For instance, one can perform statistically representative analyses up to $z \sim 1$ in the near future (e.g. KiDS de Jong et al. 2012, VIPERS Marchetti et al. 2012, Pan-STARRS Kaiser et al. 2002, DES¹, HSC²), and possibly up to $z \sim 2$ in a decade, e.g. with missions such as LSST³, and Euclid⁴ (Laureijs et al. 2011).

Alongside the progress in observational capabilities, theoretical modelling has also improved substantially. Numerical simulations have proven important to investigate the link between galaxy-galaxy lensing and the galaxy-dark matter connection(e.g. Tasitsiomi et al. 2004; Limousin et al. 2005; Natarajan et al. 2007; Hayashi & White 2008). Furthermore, the observed abundance, clustering and lensing signal have been successfully explained using a statistical description of the dark matter distribution in the Universe as provided by the halo model (see e.g. Cooray & Sheth 2002; van den Bosch et al. 2013) coupled to a realistic model that describes the way galaxies of different observable properties populate host haloes (see e.g. Yang et al. 2003; Cooray & Milosavljević 2005; Cooray 2006; Yang et al. 2008).

In this study, we examine the modeling of the galaxygalaxy lensing signal up to $z \sim 0.6$. To this end, we first compare a model that describes the statistical properties of galaxies at low redshift (van den Bosch et al. 2013, Cacciato et al. 2013) to existing galaxy-galaxy lensing data measured around Luminous Red Galaxies at higher redshift (Mandelbaum et al. 2012). To extend the redshift range even further, and to obtain higher precision measurements, we follow van Uitert et al. (2011) and complement the ninth data release (hereafter DR9) of the Sloan Digital Sky Survey (hereafter SDSS) with ~ 450 square degrees of high-quality imaging data from the second generation Redsequence Cluster Survey (RCS2, Gilbank et al. 2011).

This paper is organized as follows. We describe the analytical model in §2, its application to Luminous Red Galaxies in §3. We then describe the surveys and the strategy to extract the new lensing measurements in §4. Results are presented in §5. Conclusions are drawn and discussed in §6.

Throughout this paper, we adopt the most basic ('vanilla') Λ CDM cosmological model. Such Λ CDM cosmologies are described by 5 parameters: the energy densities (in terms of the critical density) of baryons, $\Omega_{\rm b}$, and cold dark matter, $\Omega_{\rm dm}$; the spectral index, n, and normalization, σ_8 , of the initial power spectrum; and the Hubble parameter, $h_{70} \equiv H_0/(70 \text{ km s}^{-1} \text{ Mpc}^{-1})$. The flat geometry implies that $\Omega_{\Lambda} = 1 - \Omega_{\rm m} = 1 - \Omega_{\rm b} - \Omega_{\rm dm}$. Throughout the paper, following the results of Cacciato et al. (2013), we assume ($\Omega_{\rm m}, \Omega_{\Lambda}, \sigma_8, h_{70}, n, \Omega_{\rm b}h^2$) =

³ http://www.lsst.org

(0.278, 0.722, 0.763, 1.056, 0.978, 0.0228). Radii and densities are in comoving units⁵. When physical units are used they are explicitly indicated with 'p-'. Furthermore, log is used to refer to the 10-based logarithm.

2 MODELLING GALAXY-GALAXY LENSING

In this section we briefly describe how model predictions for the galaxy-galaxy (hereafter g-g) lensing signal can be provided once one has a statistical description of dark matter properties (i.e. their average density profile, their abundance, and their large scale bias) complemented with a statistical description of the way galaxies of a given luminosity populate dark matter haloes of different masses (also known as halo occupation statistics). The model is identical to the one presented in van den Bosch et al. (2013) and successfully applied to SDSS in Cacciato et al. (2013, hereafter C13). Readers familiar with this model may skip this section and continue from §3 where we describe its application to Red Luminous Galaxies.

Weak gravitational lensing is sensitive to the mass distribution projected along the line-of-sight. Specifically, the quantity of interest is the excess surface density (ESD) profile, $\Delta\Sigma(R)$, given by

$$\Delta\Sigma(R, \bar{z}_{\rm le}) = \frac{2}{R^2} \int_0^R \Sigma(R', \bar{z}_{\rm le}) R' \, \mathrm{d}R' - \Sigma(R, \bar{z}_{\rm le}). \tag{1}$$

Here $\Sigma(R, \bar{z}_{le})$ is the projected surface mass density, which is related to the galaxy-dark matter cross correlation, $\xi_{gm}(r, \bar{z}_{le})$, according to

$$\Sigma(R, \bar{z}_{\rm le}) = \bar{\rho}_{\rm m} \int_0^{\omega_{\rm so}} \left[1 + \xi_{\rm gm}(r, \bar{z}_{\rm le})\right] \,\mathrm{d}\omega\,,\tag{2}$$

where the integral is along the line of sight with ω the comoving distance from the observer. The three-dimensional comoving distance r is related to ω through $r^2 = \omega_{le}^2 + \omega^2 - 2\omega_{le}\omega\cos\theta$. Here, ω_{le} is the comoving distance to the lens, and θ is the angular separation between lens and source (see Fig.1 in Cacciato et al. 2009). Note that the galaxy-dark matter cross correlation is evaluated at the *average* redshift of the lens galaxies, \bar{z}_{le} .

Observationally the ESD profile is inferred by measuring the average tangential distortion of background galaxies (sources) around foreground galaxies (lenses):

$$\langle \gamma_{\rm t} \rangle(R) = \frac{\Delta \Sigma(R)}{\Sigma_{\rm crit}},$$
(3)

where $\langle ... \rangle$ indicates the azimuthal average inside an annulus at distance R from the centre of the lens and of width dR. In Eq.(3), $\Sigma_{\rm crit}$ is a geometrical factor determined by the distances of (lens and source) galaxies:

$$\Sigma_{\rm crit} = \frac{c^2}{4\pi G} \frac{D_{\rm so}}{D_{\rm le} D_{\rm le-so} (1+z_{\rm le})^2},\tag{4}$$

with $D_{\rm le}$, $D_{\rm so}$, and $D_{\rm le-so}$ the angular diameter distance to the lens, the source, and between the lens and the source, respectively, and the factor $(1 + z_{\rm le})^{-2}$ accounts for our use of comoving units.

⁵ We write the mean density of the Universe as $\bar{\rho}_{\rm m} = \Omega_{\rm m} \rho_{\rm crit}$.

¹ https://www.darkenergysurvey.org

² http://www.naoj.org/Projects/HSC/HSCProject.html

⁴ http://www.euclid-ec.org

Under the assumption that each galaxy resides in a dark matter halo, $\Delta\Sigma(R, z)$ can be computed using a statistical description of how galaxies are distributed over dark matter haloes of different mass (see e.g. van den Bosch et al. 2013). Specifically, it is fairly straightforward to obtain the two-point correlation function, $\xi_{\rm gm}(r, z)$, by Fourier transforming the galaxy-dark matter power-spectrum, $P_{\rm gm}(k, z)$, i.e.

$$\xi_{\rm gm}(r,z) = \frac{1}{2\pi^2} \int_0^\infty P_{\rm gm}(k,z) \frac{\sin kr}{kr} \, k^2 \, \mathrm{d}k \,, \tag{5}$$

with k the wavenumber. $P_{\rm gm}(k, z)$, can be expressed as a sum of a term that describes the small scales (one-halo, 1h), and one that describes the large scales (two-halo, 2h), each of which can be further subdivided based upon the type of galaxies (central or satellite) that contribute to the power spectrum, i.e.,

$$P_{\rm gm}(k) = P_{\rm cm}^{\rm 1h}(k) + P_{\rm sm}^{\rm 1h}(k) + P_{\rm cm}^{\rm 2h}(k) + P_{\rm sm}^{\rm 2h}(k) \,. \tag{6}$$

As shown in van den Bosch et al. (2013), these terms can be written in compact form as

$$P_{\rm xy}^{\rm 1h}(k,z) = \int \mathcal{H}_{\rm x}(k,M,z) \,\mathcal{H}_{\rm y}(k,M,z) \,n_{\rm h}(M,z) \,\mathrm{d}M,\qquad(7)$$

$$P_{\rm xy}^{\rm 2h}(k,z) = \int dM_1 \,\mathcal{H}_{\rm x}(k,M_1,z) \,n_{\rm h}(M_1,z)$$
$$\int dM_2 \,\mathcal{H}_{\rm y}(k,M_2,z) \,n_{\rm h}(M_2,z) \,Q(k|M_1,M_2,z) \,, \qquad (8)$$

where 'x' and 'y' are either 'c' (for central), 's' (for satellite), or 'm' (for matter), $Q(k|M_1, M_2, z)$ describes the power spectrum of haloes of mass M_1 and M_2 , and it contains the large scale bias of haloes as well as a treatment of halo exclusion. Furthermore, $n_h(M, z)$ is the halo mass function of Tinker et al. 2010 (see van den Bosch et al. 2013; Cacciato et al. 2013, for further detail). Here, we have defined

$$\mathcal{H}_{\rm m}(k,M,z) = \frac{M}{\bar{\rho}_{\rm m}} \,\tilde{u}_{\rm h}(k|M,z)\,,\tag{9}$$

$$\mathcal{H}_{\rm c}(k,M,z) = \mathcal{H}_{\rm c}(M,z) = \frac{\langle N_{\rm c}|M\rangle}{\bar{n}_{\rm g}(z)},\tag{10}$$

and

$$\mathcal{H}_{\rm s}(k,M,z) = \frac{\langle N_{\rm s}|M\rangle}{\bar{n}_{\rm g}(z)} \,\tilde{u}_{\rm s}(k|M,z)\,. \tag{11}$$

Here $\langle N_{\rm c}|M\rangle$ and $\langle N_{\rm s}|M\rangle$ are the average number of central and satellite galaxies in a halo of mass $M \equiv 4\pi (200\bar{\rho}) R_{200}^3/3$, whereas $\bar{n}_{\rm g}(z)$ is the number density of galaxies at redshift z. We compute these quantities using the following expressions:

$$\langle N_{\mathbf{x}}|M\rangle = \int_{L_{-}}^{L_{+}} \Phi_{\mathbf{x}}(L|M) \,\mathrm{d}L, \qquad (12)$$

where $\Phi_{\mathbf{x}}(L|M)$ is the conditional luminosity function (see below and Appendix A), L_{-} and L_{+} refer to the lower and upper limit of a luminosity bin, respectively. Again, the subscript 'x' stands for either 'c' (centrals) or 's' (satellites), and

$$\bar{n}_{\rm g}(z) = \int \langle N_{\rm g} | M \rangle n_{\rm h}(M, z) \mathrm{d}M \,. \tag{13}$$

Furthermore, $\tilde{u}_{s}(k|M)$ is the Fourier transform of the

normalized number density distribution of satellite galaxies that reside in a halo of mass M, and $\tilde{u}_{\rm h}(k|M)$ is the Fourier transform of the normalized density distribution of dark matter within a halo of mass M. In this paper, supported by the results of Cacciato et al. (2013), we assume for both these profiles the functional form suggested in Navarro, Frenk & White (1997). The conditional luminosity function $(\Phi_{\rm x}(L|M), \text{hereafter CLF})$ describes the *average* number of galaxies with luminosities in the range $L \pm dL/2$ that reside in a halo of mass M. Following Cacciato et al. (2013), we parametrize the CLF with nine parameters (see Appendix A for a thorough description). We note here that the CLF methodology describes the halo occupation statistics of both central and satellite galaxies and it is not limited to the choice of specific luminosity bins, rather it applies to galaxies as a function of their luminosity. This will be of crucial importance when we will interpret the data presented in §4.

2.1 Additional lensing terms

In the analytical model used by C13, which was summarized above, the lensing signal is modelled as the sum of four terms: two describing the small (sub-Mpc) scale signal mostly due to the dark matter density profile of haloes hosting central and satellite galaxies; and the other two describing the large (several Mpc) scale signal due to the clustering of dark matter haloes around central and satellite galaxies, respectively. This reads:

$$\Delta\Sigma(R, z) = \Delta\Sigma_{\rm cm}^{\rm in}(R, z) + \Delta\Sigma_{\rm sm}^{\rm in}(R, z) + \Delta\Sigma_{\rm cm}^{\rm 2h}(R, z) + \Delta\Sigma_{\rm sm}^{\rm 2h}(R, z).$$
(14)

In the halo model the small scale signal (the 1-halo term) has two more contributors corresponding to: i) the baryonic mass of the galaxies themselves; and ii) the dark matter density profile of the sub-haloes which host satellite galaxies.

The smallest scales probed by the data in this study are about 50 kpc, which are much larger than the typical extent of the baryonic content of a galaxy. Therefore, it is adequate to model the lensing signal due to the baryonic content of the galaxy as the lensing due to a point source of mass $M_{\rm g} \approx M_{\rm star}$ (see e.g. Leauthaud et al. 2010). This reads

$$\Delta \Sigma^{\rm 1h,g}(R,z) \approx \frac{\langle M_{\rm star}(z_{\rm le}) \rangle_{L_{-}}^{L_{+}}}{\pi R^2} \,. \tag{15}$$

When accounting for the baryonic mass, this term adds to the other four indicated in eq.(14). Throughout the paper, we model the lensing signal as in §2. However, when describing Figure 5, we comment on how model predictions are modified once the baryonic mass is taken into account in the simplified way described above. To that aim, we use the value of the average stellar masses, $\langle M_{\text{star}} \rangle$, for the galaxies in the luminosity bins under investigation here (see §5). For completeness, we list these values in Table 1.

The modeling of the dark matter density profile of the sub-haloes which host satellite galaxies is conceptually simple (see e.g. Mandelbaum et al. 2005; Li et al. 2009; Giocoli et al. 2010; Li et al. 2012; Rodriguez-Puebla et al. 2013) However, a proper implementation of this term is hampered by the poor knowledge of the subhalo mass function (see e.g. Giocoli, Tormen & van den Bosch 2008) and of

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the stripping mechanism (see e.g. Gao et al. 2004) which occurs once a dark matter halo enters a larger halo, i.e. when an initially central galaxy becomes a satellite. Many of the results about such subhalo properties are obtained from pure N-body simulations for which the limited mass resolution may still be a important limiting factor. Furthermore, it is unclear how these results are affected by various baryonic processes in place during galaxy evolution (e.g. van Daalen et al. 2011). Given these uncertainties and since subhaloes only contribute a small fraction to the total lensing signal on small scales (see e.g. Li et al. 2009), in this paper, we refrain from modelling the lensing term due to the subhaloes which host satellite galaxies. We comment on the impact of this simplification when comparing model predictions with actual measurements.

3 SDSS LENSING SIGNAL AROUND LRGS

The model summarized in §2 was used by C13 to fit the galaxy-galaxy lensing signal measurements performed via SDSS in the spatial range $0.05 \lesssim R \lesssim 2$ Mpc and at redshift $z \lesssim 0.2$. The same model can be used to make predictions about the scale and redshift dependence of the lensing signal. To test the robustness of the model, it is therefore interesting to examine how it performs, without any adjustments of the parameters (including the best-fit cosmology from C13), when compared to different data.

We first consider the g-g lensing signal for a sample of Luminous Red Galaxies (LRGs, Eisenstein et al. 2001). Mandelbaum et al. (2012) have measured the lensing signal around two LRG samples based on the SDSS DR7 catalogue. The selection of LRGs allows the study of the dark matter distribution via weak gravitational lensing at higher red-shift compared to the main sample. The effective redshifts of the two samples are $z_{le} \approx 0.26$ and $z_{le} \approx 0.40$. Both samples have absolute magnitude limits $-23.2 < M_g < -21.1$. Note that k-corrections and evolution corrections to convert r-band magnitude to M_g are taken from Eisenstein et al. (2001). More details about the procedure to select LRGs can be found in Kazin et al. (2010) and in Mandelbaum et al. (2012).

Figure 1 shows the data (filled circles with error bars). The model predictions (solid lines) are obtained by using the same cuts as Mandelbaum et al. (2012) and the same model parameters found in C13. The model, although constrained using the main sample, describes the observed LRG signals very well. The value of the reduced χ^2 computed in the range⁶ $0.2 \lesssim R \sim 90h_{70}^{-1}$ Mpc is 1.0 for the main LRGs and 0.8 for the high-z LRGs. The lensing signal around LRGs is reproduced over a large range of scales ($0.2 \lesssim R \lesssim 90h_{70}^{-1}$ Mpc) and at high redshifts $z \sim 0.26$ and 0.4.

It is worth emphasizing that the lensing signal on small and large scales carries different information. To first order, smaller scales probe the mass distribution within haloes, whereas larger scales probe the cosmological framework (mostly through a combination of the parameters $\Omega_{\rm m}$ and σ_8). We recall here that Cacciato et al. (2013)



Figure 1. The excess surface density of LRGs from the analysis of SDSS DR7 by Mandelbaum et al. (2012). Solid lines refer to the model predictions for the lensing signal using the model parameters retrieved in C13, without further adjustment. Note that the lensing measurements are uncertain at scales smaller than about 0.14 h_{70}^{-1} Mpc as indicated by the dotted vertical line.

embedded their analysis in a fully Bayesian framework in which they also constrained the cosmological parameters which define a 'vanilla' ΛCDM cosmology. They found that the parameters $(\Omega_{\rm m}, \Omega_{\Lambda}, \sigma_8, h_{70}^{-1}, n, \Omega_{\rm b}h^2) =$ (0.278, 0.722, 0.763, 1.056, 0.978, 0.0228) best fit their model. Hence the agreement with the measurements is an important validation of the model determined by C13. It not only implies that the parameters that describe the halo occupation distribution are also valid at higher redshifts, but also that the cosmological parameters are consistent.

Before comparing the model to g-g lensing measurements based on a different data set in §4, we exploit the quality of the agreement between LRGs lensing data and model predictions to compute the average host halo mass of LRGs for both the main and the high-z sample. The estima-

 $^{^6~}$ Note that the LRG lensing measurements used here are uncertain at scales smaller than about 0.14 h_{70}^{-1} Mpc (R. Mandelbaum private communication)



Figure 2. Left: Redshift distributions of lens and source galaxies (with arbitrary normalization). The black solid histogram show the lens distribution of the $\overline{DR9}$ sample used in this paper, whereas the blue dotted histogram refers to the lens distribution of $\overline{DR7}$ used by van Uitert et al. (2011). The red dashed histogram indicates the approximate redshift distribution of the source galaxies. Right: Distribution of absolute magnitudes of the lens galaxies. The black solid histogram refers to the entire $\overline{DR9}$ sample, whereas different lines refer to different subsamples defined via redshift cuts (see legend).

tion of the average halo mass follows from

$$\langle M_{200} \rangle \equiv \frac{1}{\bar{n}_{\rm c}(z_{\rm le})} \int \langle N_{\rm c} | M \rangle n_{\rm h}(M, z_{\rm le}) M \mathrm{d}M \,,$$
 (16)

where $n_{\rm h}(M, z_{\rm le})$ is the halo mass function (Tinker et al. 2010) at the lens redshift, and $\langle N_{\rm c}|M\rangle$ is computed via eq. (12). We find that both the main and the high-z LRGs reside in haloes with $\langle \log[M_{200}/(h_{70}^{-1}M_{\odot})]\rangle \approx 13.6$, in general agreement with independent previous studies (see e.g. Mandelbaum et al. 2006; Zheng et al. 2009).

4 RCS2 GALAXY-GALAXY LENSING SIGNAL

The weak lensing signal decreases as the lens galaxy approaches the source. This limits the g-g lensing analysis of the SDSS main spectroscopic sample (DR7) to lenses with $z\sim 0.2$ when SDSS shape measurements are used. Reliable measurements at higher redshift are only possible by targeting very luminous lens galaxies; see the case of LRGs in the previous section. A unique aspect of the SDSS is the wealth of spectroscopic information, which extends to higher redshifts as is shown in Figure 2. To improve the signal-to-noise ratio for higher redshift lenses, van Uitert et al. (2011) measured the shapes of source galaxies using imaging data of a deeper survey that overlap with the SDSS. Specifically, van Uitert et al. (2011) studied the lenses in the region of SDSS DR7 that overlaps with the second generation Redsequence Cluster Survey (RCS2; Gilbank et al. 2011). RCS2 is a 900 square degree imaging survey in three bands (q',r' and i') carried out with the Canada-France-Hawaii Telescope (CFHT) using the 1 square degree camera MegaCam. The RCS2 data are ~ 2 magnitudes deeper than the SDSS in the r'-band and the median seeing of 0.7" is roughly half that of the SDSS. Consequently the survey is well suited to improve the lensing constraints for lenses with $z \geq 0.3$, where the source distribution of the main sample in SDSS decreases significantly. In particular, it allows us to improve the lensing constraints for the most massive/luminous galaxies, which are preferentially selected at higher redshifts.

As shown in Figure 2, a major change between the spectroscopic samples of SDSS DR7 and DR9 is that high redshift (z > 0.3) LRGs were targeted as part of the Baryon Oscillation Spectroscopic Survey (BOSS; Anderson et al. 2012). As we show in this section, the lensing signal around these lenses can be determined with high precision using RCS2 shape measurements.

The SDSS DR9 (Ahn et al. 2012) overlaps with 471 RCS2 pointings. This amounts to roughly 450 square degrees (about 150 square degrees more overlap than between the RCS2 and the DR7 used in Van Uitert et al. 2011). The lens sample in the study presented here consists of all objects from the DR9 in the overlapping area that have a reliable spectroscopic redshift (according to the SciencePrimay flag) and that are spectroscopically classified as galaxies. In contrast to van Uitert et al. (2011), we do not require the DR9 objects to have a match with an object from the RCS2 catalogues, but only that they reside within the field of view of a RCS2 pointing. This leads to a lens sample of ~ 70,000 objects, four times more than the lens sample used in van Uitert et al. (2011). In the remainder of the paper, we shall refer to this sample as $\overline{DR9}$.



Figure 3. Upper panel. The ESD signal measured in Van Uitert et al. (2011). Note the limited spatial scale ($0.05 \lesssim R \lesssim 2$ *p*-Mpc) and that the error budget is, on average, between 10 and 20 %. Lower panel. The ESD measurements presented in this study. Note that the probed spatial range now extends up to $R \sim 10$ *p*-Mpc and that the error budget is below 10 % for most of the probed scales.

The redshift distribution of the lenses is shown in the left panel of Figure 2. For comparison, we also show the lens sample used in van Uitert et al. (2011), labelled as $\overline{DR7}$. The majority of lenses with redshifts z > 0.3are LRGs, whereas at lower redshifts, our lens sample consists of a mix of early-type and late-type galaxies. We do not consider these samples separately in this paper, as the halo model in use does not account for this split. The right-hand panel of Figure 2 shows the distribution of absolute magnitudes for the whole sample, and for different redshift slices. The luminosities of the lenses are computed using the *r*-band Petrosian magnitudes from the SDSS photometric catalogues, corrected for extinction using the dust maps of Schlegel et al. (1998). *K* corrections were calculated to z = 0.1 using the



Figure 4. The ESD measured in this study for three subsamples of $\overline{DR9}$: i) low (z < 0.2); ii) intermediate (0.2 < z < 0.5); and iii) high (z > 0.5) redshift. Note the high quality of the signal at both intermediate and high redshift, well beyond the regime probed by SDSS-alone studies.

KCORRECT v4_2 code (Blanton et al. 2003; Blanton & Roweis 2007). Finally, a passive luminosity evolution correction, E, was computed following Mandelbaum et al. (2012). In summary, the absolute magnitudes were computed as $^{0.1}M_r = m + DM - K(z = 0.1) + E$, where m is the apparent magnitude of a galaxy, DM is the distance modulus, K is the correction mentioned above, and E is the passive evolution correction taken from Mandelbaum et al. (2012), i.e. E = 2(z - 0.1)[1 - (z - 0.1)]. We will comment on the impact of this assumption in §5.2.1

The creation of the shape measurement catalogues for the RCS2 is detailed in van Uitert et al. (2011) and we refer the reader to it for a detailed description. Since we lack redshifts for the background galaxies, we select galaxies with $22 < m_{r'} < 24$ that have a reliable shape estimate⁷ as sources. The resulting average source density is 6.3/arcmin². The approximate source redshift distribution for the sources (left-hand panel of Figure 2) is obtained by applying identical magnitude cuts to the photometric redshift catalogues

 $^{^{7}}$ We excluded sources with an ellipticity >1, and those whose photometry was deemed unreliable (e.g. due to image artefacts or neighbouring objects) as indicated by the flags of the source extraction program SExtractor (Betrin & Arnouts 1996).

of the COSMOS field (Ilbert et al. 2009). This procedure is detailed in Appendix C.

In contrast to van Uitert et al. (2011), we do not limit our lensing measurements to individual pointings, but we include the sources from neighbouring patches when measuring the azimuthally averaged tangential shear. This has the advantage that the lensing signal-to-noise at large radii improves, due to the larger number of sources at these separations. Another advantage of including neighbouring patches is that it reduces the impact of systematic contributions to the lensing signal, as is explained in Appendix B. There we also present a detailed description of the error estimate.

To compare the statistical power between the current work and the analysis in van Uitert et al. (2011), we show the lensing signal of the respective lens samples in Figure 3. We find that the signal-to-noise of the lensing measurements improves by about 50 per cent on average. Importantly, the lensing signal is robustly measured out to larger separations (see Appendix B). We also split the lens sample in three redshift bins, and show the lensing signals for each bin in Figure 4. This illustrates that even at z > 0.5 (with $\bar{z}_{le} \sim 0.59$), we are able to obtain significant lensing measurements. Furthermore, the higher normalisation of the lensing signal measured at higher redshift is indicative of the fact that, not surprisingly, more massive lenses are selected at higher redshift.

5 RESULTS

The C13 model was constrained combining SDSS galaxy abundance and clustering measurements to g-g lensing data at low redshift, $z \lesssim 0.2$, and relatively small scales, $R \lesssim 2$ Mpc. The comparison with the LRG sample in §3 provides an important test of the model, but the lensing measurements are derived from the same pipeline as the data used by C13. It is therefore interesting to study how well the predictions compare to the results of the independent analysis that uses RCS2 shape measurements. Such a comparison tests both the fidelity of the shape measurements and the model at even higher redshifts. To do so, we split the lens sample in eight luminosity bins and compare the g-g lensing signal to model predictions (see $\S5.1$). Following this comparison we proceed to use the lensing measurements to investigate the possibility to constrain the galaxy-dark matter connection at those higher redshift (see $\S5.2$).

5.1 Comparison of the RCS2 lensing signal to model predictions

To study the model predictions as a function of luminosity we divide the $\overline{DR9}$ lens sample in eight luminosity bins. The main properties of each bin are listed in Table 1. The black triangles with error bars in Figure 5 indicate the resulting ESD measurements based on the RCS2 lensing catalog as a function of projected lens-source separation. Over the large range in luminosity and scale ($0.05 \leq R \leq 10 \ h_{70}^{-1}$ Mpc) we measure a significant lensing signal. Especially the measurements for the highest luminosity bins represent an improvement over what can be done with SDSS data alone.

The magenta solid lines in Figure 5 correspond to the

Table 1. Properties of the excess surface density data

Label (1)	$ \tilde{M}_r $ (2)	lgM_{\star} (3)	\overline{z}_{le} (4)	N_{le} (5)
$egin{array}{c} L_1 \ L_2 \ L_3 \ L_4 \ L_5 \ L_6 \ L_7 \ L_8 \end{array}$	$\begin{array}{l} (-18.0,-17.0] \\ (-19.0,-18.0] \\ (-20.0,-19.0] \\ (-21.0,-20.0] \\ (-21.5,-21.0] \\ (-22.0,-21.5] \\ (-22.5,-22.0] \\ (-23.0,-22.5] \end{array}$	$\begin{array}{c} 9.21 \\ 9.72 \\ 10.23 \\ 10.75 \\ 11.09 \\ 11.32 \\ 11.57 \\ 11.82 \end{array}$	$\begin{array}{c} 0.07 \\ 0.09 \\ 0.12 \\ 0.19 \\ 0.36 \\ 0.44 \\ 0.51 \\ 0.59 \end{array}$	$\begin{array}{c} 1,418\\ 3,650\\ 8,918\\ 15,254\\ 14,013\\ 13,555\\ 5,730\\ 1517\end{array}$

The galaxy samples used to measure the excess surface density profiles, $\Delta\Sigma(R)$. For each of these samples column (1) lists the magnitude label, column (2) lists the magnitude range, where $\tilde{M}_r \equiv {}^{0.1}M_r$, column (3) lists the log of the average stellar mass $(\lg M_\star \equiv \log [\langle M_{\rm star}/h_{70}^{-1}M_\odot \rangle])$, column (4) lists the mean redshift, and column (5) lists the number of lens galaxies.

model predictions based on the study by C13. Those predictions are in overall agreement with the RCS2 measurements of the ESD on all scales, including those well outside the range used to constrain the C13 model. The agreement between model predictions and data supports the findings in C13 both in terms of the halo occupation statistics and the cosmological parameters. Furthermore, the mutual agreement of the model with the data obtained with SDSS alone (Mandelbaum et al. 2006) and with those obtained here using RCS2 implies overall consistency of the lensing signal from the two surveys. Interestingly, for the highest four luminosity bins, the C13 model predictions seem to systematically over-estimate the g-g lensing signal at the 1σ level. Although this aspect may be regarded as marginal given that the model predictions were not at all tuned to reproduce these data, we will further investigate the relevance of this slight disagreement in $\S5.2$.

Model predictions based on C13 can be easily modified to account for the contribution to the lensing signal due to the galaxy baryonic mass (red dashed lines in Figure 5). Using the simplest assumption for how the stellar content of the galaxy may contribute to the lensing signal (see §3), we note that fainter/less massive galaxies are virtually unaffected by such a correction, whereas brighter/more massive galaxies exhibit a boost of the lensing signal on scales $R \lesssim 0.2h_{70}^{-1}$ Mpc, reaching up to a factor of about 1.5 on scales $R \sim 0.05h_{70}^{-1}$ Mpc.

We note here that in the current analysis the average stellar mass per luminosity bin is estimated by matching our lens sample to the MPA-JHU DR7 value added catalogue⁸ which provides stellar mass estimates. Specifically, we use the matching objects to fit a linear relation between absolute magnitude and stellar mass, use this relation to assign a stellar mass to all our lenses, and finally determine the average for each luminosity bin (reported in Table 1). A technical caveat must be mentioned here: the MPA-JHU catalogue only contains galaxies from the DR7, and not the more recently observed ones from BOSS. Therefore, when we use the average stellar mass to compute the baryonic term

⁸ http://www.mpa-garching.mpg.de/SDSS/DR7/



Figure 5. The excess surface density around lenses in DR9 that overlap with RCS2. The black triangles indicate the lensing signal measured using the RCS2 imaging data. Magenta solid and red dashed lines refer to the model predictions from C13, see §2 and §3, respectively. The shaded cyan region refers to the MCMC results where we consider L_0, M_1, γ_2 and σ_{logLc} as free parameters.

in the halo model, we implicitly assume that the relation between luminosity and stellar mass is similar for the DR7 galaxies as for those that were observed as part of BOSS. This assumption may not be accurate, but the use of the stellar mass in this paper serves only to roughly quantify on which scales and by what amount the g-g lensing signal might be affected by the baryonic mass of the galaxy.

As a last cautionary note, we comment here on the fact that the model presented in this paper does not account for the mass distribution in the subhaloes which host satellite galaxies. As the quality of the lensing signal improves, it will become mandatory to add this extra term especially if one aims to retrieve the amount of stellar mass by fitting the small scale $(R \lesssim 0.2 h_{70}^{-1} \text{Mpc})$ lensing signal.

5.2 Constraining the galaxy-dark matter connection with weak lensing only

More luminous galaxies reside on average in more massive haloes. Using Eq.(16) we find that this is indeed the case, and that the luminosity bins listed in Table 1 correspond to halo masses that range from $\langle \log M_{200}/(h_{70}^{-1}M_{\odot}) \rangle \sim 11.4$, to $\langle \log M_{200}/(h_{70}^{-1}M_{\odot}) \rangle \sim 14.2$. The signal-to-noise of the measurements presented in Figure 5 is highest for lens galaxies with $-23 \lesssim {}^{0.1}M_r \lesssim -21$, which correspond to relatively massive haloes ($12.5 \lesssim \langle \log M_{200}/(h_{70}^{-1}M_{\odot}) \rangle \lesssim 14.2$). We explore here whether, thanks to the improved precision at the highest masses, we can constrain the model parameters which govern this regime using *solely* g-g lensing measurements. To this aim, we employ the same model used so far, but we now leave the parameters that govern the massive end of the galaxy luminosity-halo mass relation free to vary. In the parameterization used in this paper (see Appendix A for more detail), the relation between the luminosity of a central galaxy and its host halo mass is assumed to be:

$$L_{c}(M) = L_{0} \frac{(M/M_{1})^{\gamma_{1}}}{[1 + (M/M_{1})]^{\gamma_{1} - \gamma_{2}}}$$

 $\sim L_{0} \left(\frac{M}{M_{1}}\right)^{\gamma_{2}} \text{ for } M \gg M_{1}.$ (17)

Furthermore, the *average* number of central galaxies of a given luminosity is related to the halo mass via a log-normal distribution:

$$\Phi_{\rm c}(L|M) \,\mathrm{d}L = \frac{\log e}{\sqrt{2\pi} \,\sigma_{\log L_{\rm c}}} \exp\left[-\frac{(\log L - \log L_{\rm c})^2}{2 \,\sigma_{\log L_{\rm c}}^2}\right] \frac{\mathrm{d}L}{L},$$
(18)

where $\sigma_{\log L_c}$ indicates the scatter in luminosity at fixed halo mass and $\log L_c$ is, by definition, the expectation value for the logarithm of the luminosity of the central galaxy:

$$\log L_{\rm c} = \int \Phi_{\rm c}(L|M) \, \log L \, \mathrm{d}L \,. \tag{19}$$

Here, we consider L_0, M_1, γ_2 , and $\sigma_{\log L_c}$ as four free parameters, while keeping γ_1 fixed to 3.18, the value retrieved in C13. The first free parameter has the units of a luminosity $(h_{70}^{-2}L_{\odot})$, the second has the units of a mass $(h_{70}^{-1}M_{\odot})$, whereas the remaining two are dimensionless.

To determine the probability distribution of the model parameters discussed above we run a Markov Chain Monte Carlo⁹ (hereafter MCMC) using the standard Metropolis-Hasting algorithm (Metropolis et al. 1953). In this chain, the parameters L_0 , M_1 , γ_2 and $\sigma_{\log L_c}$ are free to vary and no prior information is used, whereas the remaining parameters are fixed at the same value as the one in the C13 model (see also Appendix A). As the satellite fraction is supposed to be very low for bright galaxies (e.g. Mandelbaum et al. 2006; Cacciato et al. 2009, 2013; van Uitert et al. 2011), and the faintest galaxies in this study have relatively large uncertainties, we do not expect significant biases from selecting a subsample of the model parameters that governs the galaxydark matter connection of central galaxies only.

The 95% confidence levels of the g-g lensing models explored with the MCMC are indicated by the cyan shaded regions in Figure 5. As expected, the subset of parameters that we have varied has almost no impact on the predictions for the lensing signal around the faintest galaxies. For brighter galaxies, the MCMC brings the model in better agreement with the observables than the initial C13 model predictions.



Figure 6. Posterior distribution of the parameter $\sigma_{\log L_c}$. Blue shaded histogram refers to the analysis in §5.1 and §5.2, whereas the grey shaded histogram refers to the result of the test performed in §5.2.1 to assess the sensitivity of the analysis to passive evolution correction. The value of the scatter $\sigma_{\log L_c}$ is robust against the uncertainties deriving from passive evolution correction.

The agreement between model predictions and data has improved by assigning smaller halo masses to galaxies of the same luminosity. From our analytical model (see especially eq. 17 and 18), one can see that lower halo masses at the same luminosity can be obtained by altering the $L_{\rm c}(M)$ relation at the massive end or by increasing the scatter, $\sigma_{\log L_c}$. As outcome of the MCMC we find that the $L_{\rm c}(M)$ relation has substantially changed from the one retrieved in C13. However, as discussed in the following subsection, the inference of the parameters which govern the $L_{\rm c}(M)$ relation is very sensitive to the assumed correction for luminosity evolution, which is uncertain. Interestingly, the inference of the parameter $\sigma_{\log L_c}$ is more robust against those uncertainties. Therefore, we report here only the corresponding result. The blue shaded histogram in Fig.6 shows the posterior distribution of the scatter in the number of galaxies of a given luminosity at any halo mass, σ_{logL_c} (see Appendix A for more details on this parameter). We find that $\sigma_{logL_c} = 0.146 \pm 0.011$ (median \pm one standard deviation), in excellent agreement with independent studies based on abundance, clustering, and/or satellite kinematics at lower redshift. Specifically, using a large SDSS galaxy group catalogue, Yang, Mo & van den Bosch (2008) obtained $\sigma_{\rm logL_c} = 0.13 \pm 0.03$ (black star) and they did not find evidence for a halo mass dependence. Cooray (2006) explicitly assumed no mass dependence in $\sigma_{\rm logL_c}$ when studying the luminosity function and clustering properties of SDSS galaxies, and found $\sigma_{logL_c} = 0.17^{+0.02}_{-0.01}$ (grey triangle). More et al. (2009) studied the properties of

 $^{^9}$ The chain consists of four different chains which start from different initial guesses in the parameter space. In total, we perform about three million model evaluations. With an average acceptance rate of $\sim 30\%$, the complete chain used in the analysis is a well converged chain of one million model evaluations.



Figure 7. Upper panel. The reference absolute magnitude, $^{0.1}M_r = {}^{z}M_r + E$, as a function of redshift computed via the passive evolution correction, E, suggested by Blanton et al. (2003, blue solid line labelled B03) and Mandelbaum et al. (2012, dashed red line labelled M12) for the case of a galaxy with ${}^{z}M_r = -20$. Note that a galaxy with ${}^{z}M_r = -20$ at z > 0.1 will be rescaled to a fainter reference magnitude, ${}^{0.1}M_r$, at the reference redshift z = 0.1. Lower panel. The difference between the B03 and M12 passive evolution corrections, $\Delta^{0.1}M_r = {}^{0.1}M_r^{B03} - {}^{0.1}M_r^{M12}$.

satellite galaxy kinematics around massive/luminous central galaxies and found $\sigma_{logL_c} = 0.16 \pm 0.04$ (magenta circle). Finally, C13 combining abundance, clustering and lensing of galaxies in SDSS found $\sigma_{logL_c} = 0.157 \pm 0.007$ (red square).

5.2.1 Sensitivity to passive evolution correction

Ideally, one would like to compare the results on the $L_c(M)$ relation obtained here with those obtained at lower redshift to infer an evolutionary scenario. However, the physical interpretation of the results of the MCMC is hampered by the fact that the sample of galaxies used in this analysis is not uniform. In fact, the mix of early- and late-type galaxies changes with redshift due to the luminosity-based selection of the lenses. This leads to an uncertain correction for luminosity evolution which enters in the definition of the reference absolute magnitude of a galaxy, ${}^{0.1}M_r$ (see §4). In what follows, we will show how a small variation in the correction of luminosity evolution impacts the lensing analysis and the corresponding model parameters. This example highlights the sensitivity of our model to the intrinsic luminosity evolution of a galaxy.

In the analysis presented in §5.1 and 5.2, we have used the passive evolution correction, E, suggested in Mandelbaum et al. (2012). However, Blanton et al. (2003) suggested a simpler functional form that has been widely adopted in the literature: E = 1.6(z - 0.1). The difference between these two functions is highest at higher redshift, reaching about 0.3 magnitude at the highest redshift of interest here, $z \sim 0.6$ (see Fig. 7). Using the Blanton et al.

(2003) expression for the passive evolution would have only a minor impact on the lensing analysis by Mandelbaum et al. (2012) as LRGs are selected only up to $z \sim 0.4$ and they are not split further into luminosity bins. However, in our analysis lens galaxies are selected in narrow luminosity bins and up to $z \sim 0.6$. A different selection of lens galaxies directly translates into a different lensing signal. Specifically, we find that using the passive evolution correction of Blanton et al. (2003) leaves the lensing signal of faintest galaxies virtually unaltered, while leading to a higher normalisation (at about 1σ level) of the lensing signal of the four brightest bins. Repeating the MCMC analysis as in §5.2 but on this new selection of lenses, we recover similar values of the scatter, $\sigma_{\log L_c}$, as indicated by the grey shaded histogram in Fig.6. However, we retrieve values for M_1 , L_0 , and γ_2 that differ by about 3 σ from our initial analysis¹⁰. The changes in the retrieved model parameters are to be attributed to the expected degeneracies due to the assumed parametrization (see eq. [17]). We conclude that the retrieved values of the model parameters M_1 , L_0 , and γ_2 are sensitive to the details of luminosity evolution correction. Therefore, in this paper we refrain from drawing any conclusion about the cosmic evolution of the $L_{\rm c}(M)$ relation, deferring it to the analysis of a homogeneous sample of early-type galaxies (van Uitert et al., in preparation) for which passive luminosity correction can be modelled with higher confidence than in the current study where we consider a mix of early- and late-type lens galaxies.

6 CONCLUSIONS

We investigated how measurements of the galaxy-galaxy lensing signal around lenses at increasingly higher redshifts and at larger projected distances can be used to study the galaxy-dark matter connection. We showed that the analytical model presented in van den Bosch et al. (2013) and constrained by (SDSS) abundance, clustering, and lensing data at z < 0.2 (see C13), reproduces, without further adjustments, the galaxy-galaxy lensing signal measured around Luminous Red Galaxies (Mandelbaum et al. 2012) at $\bar{z}\sim 0.26$ and $\bar{z}\sim 0.4$ and throughout the probed spatial range, $0.02 \lesssim R \lesssim 90 h_{70}^{-1}$ Mpc (see Figure 1). This agreement is an important validation of the model determined by C13. It not only implies that the parameters that describe the halo occupation distribution are also valid at higher redshifts, but also implies consistency with the cosmological parameters found by C13.

Following van Uitert et al. (2011), we measure the lensing signal around lenses from the Sloan Digital Sky Survey (Data Release 9) using shape measurements from the 450 square degrees that overlap with the second generation Red-sequence Cluster Survey (RCS2, Gilbank et al. 2011). The higher source density and redshift results in a significant improvement, compared to SDSS data alone, for lenses with $z \gtrsim 0.3$. We split the lenses into eight luminosity bins and measure robust tangential shear signals as a function of the transverse separation, R, in the range $0.05 \lesssim R \lesssim 10h_{70}^{-1}$ Mpc (see Figure 5).

 $^{10}\,$ We have checked that also this MCMC has converged.

Compared to the earlier study by van Uitert et al. (2011) which used the overlap with DR7, the use of the overlap with DR9 increases the number of lenses, resulting in an improvement of about 50% in the precision of the lensing shear over the entire range probed here $(0.05 \leq R \leq 10 \text{ Mpc})$. In addition we now include the sources from neighbouring RCS2 pointings (previously the analysis was done on a pointing-by-pointing basis). This increases the lensing signal-to-noise at large projected lens-source separations (see Figure 3), and reduces systematic contributions to the lensing signal (see Figure B1). Finally, compared to the DR7 catalogue, the redshift distribution of lens galaxies in DR9 has a large number of galaxies at z > 0.4 (see Figure 2), enabling us to probe the matter distribution at those high redshifts (see Figure 4).

We split the lens galaxies in 8 luminosity bins, ranging from $-18 < {}^{0.1}M_r < -17$ to $-23 < {}^{0.1}M_r < -22.5$. Brighter galaxies are distributed at increasingly higher redshift such that the data span a wide range in redshift from $\bar{z} = 0.07$ to $\bar{z} = 0.59$. Moreover, since brighter galaxies live on average in more massive haloes, the range in luminosity probed here spans a correspondingly wide range in host halo mass. As a result, the measurements presented here *simultaneously* probe the matter distribution in different regimes from small groups to massive clusters, and from low to high redshift (see Figure 5).

Without any adjustment, the C13 model also describes the lensing signal obtained with RCS2 data very well. This corroborates the results based on the SDSS analysis of LRGs, but also implies consistency of the measurement of the lensing signal. We note that on the smallest scales probed here $(0.05 \leq R \leq 0.2h_{70}^{-1} \text{Mpc})$, we find better agreement if we include a contribution from the stellar mass of the galaxies: a simple point-mass model for the stellar component of the galaxies is sufficient to boost model predictions at those small scales leading to a better agreement with the data.

Finally, exploiting the high signal-to-noise ratio of the lensing signal around bright galaxies, we attempt to constrain aspects of the galaxy-dark matter connection across cosmic time. While the inference of an evolutionary scenario for the galaxy luminosity-halo mass relation is hampered by current uncertainties in the evolution of galaxy luminosity, we robustly assess that, up to $z \sim 0.6$, the number of central galaxies as a function of halo mass is well described by a lognormal distribution with scatter, $\sigma_{\log L_c} = 0.146 \pm 0.011$, in agreement with previous independent studies at lower redshift.

Our results demonstrate the value of complementing the excellent information about the properties of the lenses provided by the SDDS with deeper, high-quality imaging data. This allows us to probe the link between galaxies and matter around them in increasing level of detail and at increasingly higher redshift. In this paper we tested the model of C13 and found that it overall performs very well. In future publications we will use our data to examine the evolution of early-type galaxies only, and we will carry out a comprehensive study of the possible evolution with cosmic time of the galaxy luminosity-halo mass relation for early-type galaxies.

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APPENDIX A: THE CONDITIONAL LUMINOSITY FUNCTION

Throughout the paper, the *average* number of galaxies with luminosities in the range $L \pm dL/2$ that reside in a halo of mass M is described by the conditional luminosity function, $\Phi(L|M)$, introduced by Yang et al. (2003):

$$\langle N_{\mathbf{x}}|M\rangle = \int_{L_1}^{L_2} \Phi_{\mathbf{x}}(L|M) \,\mathrm{d}L.$$
 (A1)

Following Cooray & Milosavljević (2005) and Cooray (2006), we split the conditional luminosity function (hereafter CLF) in two components,

$$\Phi(L|M) = \Phi_{\rm c}(L|M) + \Phi_{\rm s}(L|M), \qquad (A2)$$

where $\Phi_c(L|M)$ describes the contribution due to central galaxies (defined as those galaxies that reside at the center of their host halo), while $\Phi_s(L|M)$ characterizes satellite galaxies (those that orbit around a central).

Our parameterization of the CLF model is motivated by the results obtained by Yang et al. (2008) from a large galaxy group catalogue (Yang et al. 2007) extracted from the SDSS Data Release 4, and by Tal et al. (2012) from a study of the luminosity function of satellite galaxies of luminous red galaxies. In particular, the CLF of central galaxies is modeled as a log-normal function:

$$\Phi_{\rm c}(L|M)\,\mathrm{d}L = \frac{\log e}{\sqrt{2\pi}\,\sigma_{\rm logL_c}} \exp\left[-\frac{\left(\log L - \log L_{\rm c}\right)^2}{2\,\sigma_{\rm logL_c}^2}\right]\,\frac{\mathrm{d}L}{L}\,,(A3)$$

and the satellite term as a modified Schechter function:

$$\Phi_{\rm s}(L|M)\,\mathrm{d}L = \phi_{\rm s}^* \,\left(\frac{L}{L_{\rm s}^*}\right)^{\alpha_{\rm s}+1} \,\exp\left[-\left(\frac{L}{L_{\rm s}^*}\right)^2\right] \frac{\mathrm{d}L}{L}\,,\qquad(\mathrm{A4})$$

which decreases faster than a Schechter function at the bright end. Note that L_c , σ_c , ϕ_s^* , α_s and L_s^* are in principle all functions of the halo mass M.

Following Cacciato et al. (2009), and motivated by the results of Yang et al. (2008) and More et al. (2009, 2011), we assume that σ_{logL_c} , which expresses the scatter in log L of central galaxies at fixed halo mass, is a constant (i.e. is independent of halo mass and redshift). Note though that this does not imply that the scatter in halo mass at a fixed luminosity, σ_{logM} , is constant: as discussed in Cacciato et al. (2009) and More et al. (2009), σ_{logM} increases because the slope of the $L_c(M)$ relation becomes shallower with increasing M. In addition, for L_c , we adopt the following parameterization:

$$L_{\rm c}(M) = L_0 \frac{(M/M_1)^{\gamma_1}}{[1 + (M/M_1)]^{\gamma_1 - \gamma_2}}.$$
 (A5)

Hence, $L_c \propto M^{\gamma_1}$ for $M \ll M_1$ and $L_c \propto M^{\gamma_2}$ for $M \gg M_1$. Here M_1 is a characteristic mass scale, and $L_0 = 2^{\gamma_1 - \gamma_2} L_c(M_1)$ is a normalization. For the satellite galaxies we adopt

$$L_{\rm s}^*(M) = 0.562L_{\rm c}(M)\,,\tag{A6}$$

$$\alpha_{\rm s}(M) = \alpha_{\rm s} \tag{A7}$$

(i.e., the faint-end slope of $\Phi_{\rm s}(L|M)$ is independent of mass and redshift), and

$$\log[\phi_{\rm s}^*(M)] = b_0 + b_1(\log M_{12}) + b_2(\log M_{12})^2, \qquad (A8)$$

with $M_{12} = M/(10^{12} h_{70}^{-1} \text{ M}_{\odot})$. Note that neither of these functional forms has a physical motivation; they merely were found to adequately describe the results obtained by Yang et al. (2008) from the SDSS galaxy group catalogue.

To summarize, our parameterization of the CLF thus has a total of nine free parameters. Based on the results of Cacciato et al. (2013), unless otherwise specified, we adopt the values $(\log[M_1/(h_{70}^{-1}M_{\odot})], \log[L_0/(h_{70}^{-2}L_{\odot})], \gamma_1,$ $\gamma_2, \sigma_{\log L_c}, \alpha_s, b_0, b_1, b_2)=(11.39, 10.25, 3.18, 0.245, 0.157,$ -1.18, -1.17, 1.53,-0.217).

APPENDIX B: SHEAR SYSTEMATICS AND COVARIANCE MATRIX

For this paper the inaccuracies in the correction for PSF anisotropy is a main source of bias. On small scales we average over many lens-source pairs which have a random orientation with respect to the direction of PSF anisotropy, and as a result the lensing signal is robust. At large radii the lensing signal is small and residual systematics may become more dominant because the angles between the PSF anisotropy and the lens-source pairs may no longer be isotropic because of masks or the survey geometry. In this section we therefore examine the reliability of the lensing signal on scales $\gtrsim 1h_{70}^{-1}p-{\rm Mpc}.$

To account for residual systematics in our shape measurement catalogues, and for the impact of image masks on the lensing signal, we compute the lensing signal around a large number of random points and subtract that from the galaxy-galaxy lensing signal. In the absence of systematics or an isotropic orientation of lens-source pairs, this signal should vanish. The red line in Figure B1 compares this signal to the observed lensing signal (triangles with error bars) for the various luminosity bins. We find that the correction is generally very small: it is negligible for the L1 to L5 bins, and for the other bins it is significantly smaller than the lensing signal over the range $0.05 < r < 10 \ h_{70}^{-1} p$ -Mpc which we use in our analysis. Therefore, any small error in the calculation of the random shear signal will have a minor effect at most on our results.

The random shear signal is small because we include neighbouring pointings in our lensing analysis. Consequently, at large projected separations, the lensing signal is averaged over many more lens-source orientations, which averages out any residual systematics in our shape measurement catalogues at those scales. That it is important to include the neighbouring pointings, as opposed to analysing each pointing separately, is demonstrated by the dashed orange curves: these show the random signal for the case where we perform the lensing measurements on single exposures only. In this case, the random signal strongly increases with increasing lens-source separation, and its amplitude becomes comparable or even larger than the size of the lensing signal itself. Even a relatively small error in the computation of the random lensing signal could seriously affect the results, which is clearly undesirable. Therefore, it is important to conduct the analysis on patches rather than individual exposures.

Another commonly used test is the measurement of the cross-shear, which is the projection of the source ellipticities to a unit vector that is rotated by 45 degrees from the vector that connects that lens and the source. Galaxygalaxy lensing only produces tangential shear and not cross shear, as the average gravitational potential of a large number of lenses with random orientations is circularly symmetric. Therefore, if we measure a cross shear that is not consistent with zero this indicates the presence of systematics in our shape measurement catalogues. For all luminosity bins we find that the cross-shear is consistent with zero on all scales used in our analysis.

In order to quantify the level of correlation between the lensing signals of different radial bins, we compute the correlation matrix from the data using the "delete one jackknife" method (Shao 1986). We treat each pointing of the total N pointings as a sub-volume that is subsequently left out to create a new realization of the data. The covariance matrix is then determined as

$$C_{ij}(\gamma_i, \gamma_j) = \frac{N-1}{N} \sum_{k=1}^{N} (\gamma_i^k - \bar{\gamma}_i) (\gamma_j^k - \bar{\gamma}_j), \qquad (B1)$$

with γ_i the shear of the *i*-th radial bin, γ_i^k the shear at that location from one of the jackknife realizations, and $\bar{\gamma}_i$ the mean shear of that bin determined by averaging over all the jackknife realizations. The correlation matrix follows from $\operatorname{Corr}_{ij} = C_{ij}/\sqrt{C_{ii}C_{jj}}$.

We find that the correlation matrix is practically diagonal for almost all of our luminosity bins. Only for L2, L3 and L4 we find some low-level off-diagonal terms only around scales of $\sim 1 h_{70}^{-1} p$ -Mpc. Since our analysis is mostly sensitive to the highest luminosity bins, we assume that the correlation matrices are diagonal when we fit the models to the data.

Note that the correlation matrix that results from the jackknife method depends on the size of the sub-volume



Figure B1. Assessment of systematics in the excess surface density measurements. Each panel refer to a luminosity bin as indicated by the labels. Black triangles with error bars refer to the excess surface density signal. The orange dashed lines denote the random shear signal obtained if one were to use only one single exposure, whereas the red solid line refers to the random shear signal obtained including neighbouring patches as in our analysis.

that is subsequently left out. This is demonstrated in Norberg et al. (2008), who compared several ways to determine the variance and covariance of 2-point clustering measurements. For our purposes, the covariance matrix we determine is expected to be sufficiently accurate. However, for using measurements like these to constrain cosmological parameters, this is an issue that needs to be addressed, separately from the effect that the inverse of a noisy but unbiased correlation matrix is not unbiased (Hartlap et al. 2007).

APPENDIX C: SOURCE REDSHIFT DISTRIBUTION

Using lens galaxies at higher redshifts, the mean lensing efficiency $\langle D_{\rm ls}/D_{\rm s} \rangle$ becomes more sensitive to the adopted redshift distribution of the sources, $P(z_{so})$. Therefore, we have updated the method for determining $P(z_{so})$. van Uitert et al. (2011) used the photometric redshift catalogues of the CFHTLS "Deep Survey" fields (Ilbert et al. 2006) and selected all objects in the range 22 < r' < 24that satisfied the selection cuts as described in the release notes that accompanied the catalogues, i.e. only objects with reliable photometry in all the bands, that were observed in unmasked regions and with a best-fit template number < 54. Since the main interest there was to determine the redshift distribution rather than to select galaxies with reliable photometric redshifts, galaxies in the redshift range $0.05 < z_{\rm phot} < 2.0$ were selected instead of $0.2 < z_{\rm phot} < 1.5$ where the redshifts were deemed reliable. van Uitert et al. (2011) did not account for the scatter of the photometric redshifts, nor for the fraction of outliers. Also, they did not account for the fact that bright sources have a larger weight in the lensing measurements than faint ones.

To increase the precision of the lensing efficiencies for higher lens redshifts, in this paper, we use the photometric redshift catalogue from the 2 deg² COSMOS field (Ilbert et al. 2009) instead. The photometry in 30 bands results in photometric redshifts that are both more accurate than those from the CFHTLS, and also more reliable up to higher redshifts. Using the overlap with the CFHTLS-D2 catalogue, kindly provided by H. Hildebrandt, we determined the conversion between the r^+ -band from the COS-MOS catalogues, and the r'-band from the CFHTLS. Using this conversion we selected source galaxies in COSMOS based on their r^+ magnitudes corresponding to a selection of 22 < r' < 24.

When integrating $D_{\rm ls}/D_{\rm s}$ over the $P(z_{\rm so})$, we have to account for the fact that bright galaxies have a larger weight in our lensing measurements than faint ones. For this purpose, we determined the average lensing weight of the source galaxies in the RCS2 in narrow r'-band magnitude bins, finding that on average the sources with $r' \sim 22$ have a weight that is twice that of $r' \sim 24$ source galaxies. We used the conversion between the r^+ - and r'-band to compute the corresponding weight of each galaxy in the COSMOS catalogue, and used that weight to determine the weighted mean lensing efficiency.

To account for the outliers, we assigned a new redshift to a random fraction of the galaxies equal to the outlier fraction. The new redshift was drawn from the photometric redshift distribution of the sources, and replaced the catalogue value when it fulfilled the outlier criterion

 Table C1. Lensing efficiencies determined from the photometric redshift distributions of source galaxies

$\binom{z_{\text{lens}}}{(1)}$	$ \begin{array}{c} \langle D_{ls}/D_s \rangle \\ (2) \end{array} $	$ \begin{array}{c} \langle D_{ls}/D_s \rangle \text{ (CFHT)} \\ (3) \end{array} $	$ \begin{array}{c} \langle D_{ls}/D_s \rangle \; (z_{\rm phot} < 2) \\ (4) \end{array} $
0.1	0.774	0.777(1.00)	0.768
0.2	0.602	$0.586\ (1.03)$	0.588
0.3	0.461	0.440(1.05)	0.443
0.4	0.350	0.328(1.07)	0.329
0.5	0.264	0.240(1.10)	0.241
0.6	0.194	0.172(1.13)	0.172
0.7	0.141	0.118(1.19)	0.118
0.8	0.103	0.079(1.30)	0.081
0.9	0.076	0.053(1.43)	0.055
1.0	0.057	0.036(1.58)	0.037

(1) lens redshift; (2) lensing efficiency determined using the photometric redshift catalogues of COSMOS (Ilbert et al. 2009); (3) lensing efficiency determined using the photometric redshift catalogues of the CFHTLS "Deep Survey" fields (Ilbert et al. 2006), restricted to source galaxies in the range $z_{\rm phot} < 2$. The bracketed values show the ratio between column 2 and 3; (4) lensing efficiency determined using the photometric redshift catalogues of COSMOS, restricting the source galaxies in the range $z_{\rm phot} < 2$.

 $|z_{\rm random} - z_{\rm phot}|/(1 + z_{\rm random}) > 0.15$. The outlier fraction depends on the brightness; we adopted a value of 0.7% for galaxies with $i^+ < 23$, and 15.3% for galaxies with $i^+ > 23$, as quoted in Ilbert et al. (2009). We created 16 realizations of the photometric redshift catalogues, each with a different randomly assigned set of outliers, and adopted the mean as our new lensing efficiencies. The scatter between the different realizations is small, and can safely be ignored compared to the statistical errors of the lensing analysis. We show the mean lensing efficiency at 10 lens redshifts in the second column of Table C.

We ignored the impact of scatter of the photometric redshifts with respect to the spectroscopic redshifts. The effect of scatter is that it moves galaxies in redshift from where their abundance is large to where it is small. To estimate the impact that might have on $\langle D_{\rm ls}/D_{\rm s} \rangle$, we additionally scattered each photometric redshift by randomly drawing a value from a Gaussian, whose width depends on the galaxies' *i*-band magnitude, as quoted in Ilbert et al. (2009). We multiplied that random value with $1 + z_{\rm phot}$ and added it to $z_{\rm phot}^{11}$. We created 16 new realizations, and determined the mean lensing efficiency. We found that the impact is less than a percent at all lens redshifts, and can therefore be safely ignored.

To see how the lensing efficiencies compare to those computed using the CFHTLS "Deep" catalogues, we applied the same procedure to compute the average lensing efficiencies. We accounted for outliers by adopting the outlier fractions as a function of i'-band magnitude from Ilbert et al. (2006), and applied the same weight as a function of r'-band magnitude. However, we only selected galaxies with $z_{\rm phot} < 2$. Again, we created 16 realizations, and determined the mean. We show the resulting values of $\langle D_{\rm ls}/D_{\rm s} \rangle$ in the third column of Table C. At low redshifts, the resulting lensing efficiencies only differ by a few percent compared to the ones based on the COSMOS catalogue. However, we find that if the lens redshift increases, the $\langle D_{\rm ls}/D_{\rm s} \rangle$ from COS-MOS becomes increasingly larger. To demonstrate that this difference is due to source galaxies at $z_{\rm phot} > 2$, we repeated the calculation using the COSMOS photometric redshift catalogue, but now restricting the analysis to $z_{\rm phot} < 2$. We show the resulting lensing efficiencies in the fourth column of Table C. We find that the lensing efficiencies agree very well with those based on the CFHTLS "Deep" catalogues. The difference is at most 4% over the entire redshift range that we probed.

In previous work where we used the photometric redshift catalogues from Ilbert et al. (2006) to compute the lensing efficiencies, we focused at galaxies at low redshifts. Hence the lensing efficiencies that we used there were of sufficient accuracy. However, for galaxies at redshifts z > 0.5, our results show that is it important to include source galaxies at $z_{\rm phot} > 2$ in the computation of $\langle D_{ls}/D_s \rangle$.

Note that we have ignored cosmic variance. However, we find very similar lensing efficiencies using the COSMOS and CFHTLS "Deep" photometric redshift catalogues when we restrict the galaxies to $z_{\rm phot} < 2$. This suggests that cosmic variance does not have a large impact on the lensing efficiencies that we use.

¹¹ Formally, Ilbert et al. (2009) quote the scatter on $\Delta z/(1 + z_{\rm spec})$ with $\Delta z = z_{\rm phot} - z_{\rm spec}$, so we should have multiplied the random value with $1 + z_{\rm spec}$ rather than $1 + z_{\rm phot}$. However, we expect the difference to be minor.