

CONSTRAINING THE CENTRAL MAGNETIC FIELD OF MAGNETARS

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The magnetars are believed to be highly magnetized neutron stars having surface magnetic field $10^{14} - 10^{15}$ G. It is believed that at the center, the magnetic field may be higher than that at the surface. We study the effect of the magnetic field on the neutron star matter. We model the nuclear matter with the relativistic mean field approach considering the possibility of appearance of hyperons at higher density. We find that the effect of magnetic field on the matter of neutron stars and hence on the mass-radius relation is important, when the central magnetic field is atleast of the order of 10^{17} G. Very importantly, the effect of strong magnetic field reveals anisotropy to the system. Moreover, if the central field approaches 10^{19} G, then the matter becomes unstable which limits the maximum magnetic field at the center of magnetars.

Keywords: neutron star; hyperon matter; magnetic field; magnetar

1. Introduction

Anomalous X-ray pulsars and soft γ -ray repeaters are observationally identified with highly magnetized neutron stars, known as magnetars, with surface magnetic field $\sim 10^{14} - 10^{15}$ G.¹ The processes of supernova collapse will leave behind a strongly non-uniform frozen-in field distribution. Also any dynamo mechanism generating fields will carry the imprint of inhomogeneous density profile in the star. Thus, to maintain the local magneto-static equilibrium, more realistic treatment of the equation of state (EoS) of matter for a magnetar requires inclusion of gradually increasing magnetic field from surface to center. Massive compact stars are likely to develop exotic cores with one possibility of appearance of hyperons with the increasing density. In the present work, considering the radial profile of the magnetic field and carefully analyzing the different components of the field, we show that the pressure of the magnetar matter parallel to the magnetic field exhibits instability.

2. Model of magnetar matter

To construct the model of dense matter, we employ non-linear Walecka mean field theory^{2,3} of nuclear matter including the possibility of appearance of hyperons and muons at higher density. In the presence of magnetic field, the Lagrangian density of the system is $\mathcal{L} = \sum_b \mathcal{L}_b + \sum_l \mathcal{L}_l - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$, with \mathcal{L}_b and \mathcal{L}_l are the baryonic and leptonic Lagrangian densities respectively in the presence of magnetic field^{4,5} and $F_{\mu\nu}$ the electro-magnetic field tensor. For details, see Ref. 6.

Total energy density and pressure of the system can be obtained by considering the energy-momentum tensor of the system $T^{\mu\nu} = T_m^{\mu\nu} + T_f^{\mu\nu}$, where $T_m^{\mu\nu}$ and $T_f^{\mu\nu}$

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are the matter and field parts respectively. In the absence of electric field, and in the rest frame of fluid

$$T^{\mu\nu} = \begin{bmatrix} \varepsilon_m + \frac{B^2}{8\pi} & 0 & 0 & 0 \\ 0 & P - MB + \frac{B^2}{8\pi} & 0 & 0 \\ 0 & 0 & P - MB + \frac{B^2}{8\pi} & 0 \\ 0 & 0 & 0 & P - \frac{B^2}{8\pi} \end{bmatrix}, \quad (1)$$

when the magnetic field is considered to be along z -direction with $B^\mu B_\mu = -B^2$, B is the magnitude of magnetic field, M the magnetization per unit volume, P and ε_m are respectively the pressure and energy density of the matter. This clearly shows anisotropic nature of the pressure in the presence of (strong) magnetic field. ε_m is calculated using the charged single particle energy $E_n = \sqrt{p_z^2 + m^2 + 2ne|Q|B}$, considering the quantized phase space in the presence of magnetic field, where p_z is the component of momentum along z -axis, m the mass, $e|Q|$ the total charge, with e being the electron's charge, of the particle, n is the number of occupied Landau level. Then matter pressure is $P = \sum_b \mu_b n_b + \sum_l \mu_l n_l - \varepsilon_m$, where $\mu_{b,l}$ and $n_{b,l}$ are respectively the chemical potentials and number densities for baryons (b) and leptons (l). The density profile of the magnetic field is modeled as⁷

$$B\left(\frac{n_b}{n_0}\right) = B_s + B_c \left\{ 1 - \exp\left[-\beta\left(\frac{n_b}{n_0}\right)^\gamma\right] \right\}, \quad (2)$$

where β and γ are two parameters, n_b and n_0 are respectively the number densities of matter and nuclear matter, B_s and B_c are respectively the magnitudes of the magnetic field in the surface and center of the underlying magnetar.

3. Results

Figure 1a shows EoS for hypernuclear matter in strong and fixed magnetic field profiles. For non-zero magnetic field, the pressure splits into the parallel (P_{\parallel}) and transverse (P_{\perp}) components, and exhibits anisotropy. It is seen that the low-density behavior of EoS with constant magnetic field implies unrealistically large anisotropic magnetic field up to the surface of the star, which is inconsistent with the inferred surface magnetic field ($\sim 10^{15}$ G) of magnetars. It is seen from Fig. 1b that for a given value of β , the EoS becomes softer with increasing γ . Consequently, beyond a certain critical γ and in a certain density regime, P ceases to increase (and eventually decreases) with the further increase in n_b . This implies the onset of instability of matter above that value of density for that particular B_c and magnetic field profile. We also show the results for each β with $\gamma = 1$ (minimum value). Note that the maximum γ is taken in such a way that P forms a plateau as a function n_b . Furthermore, it is evident that with the decrease of β , the instability occurs at larger values of γ and n_b .

The instability arises due to the negative contribution from the field energy density to the pressure of magnetized baryons and leptons in the direction of the magnetic field, which is evident from Eq. (1). With the increase of n_b , more negative

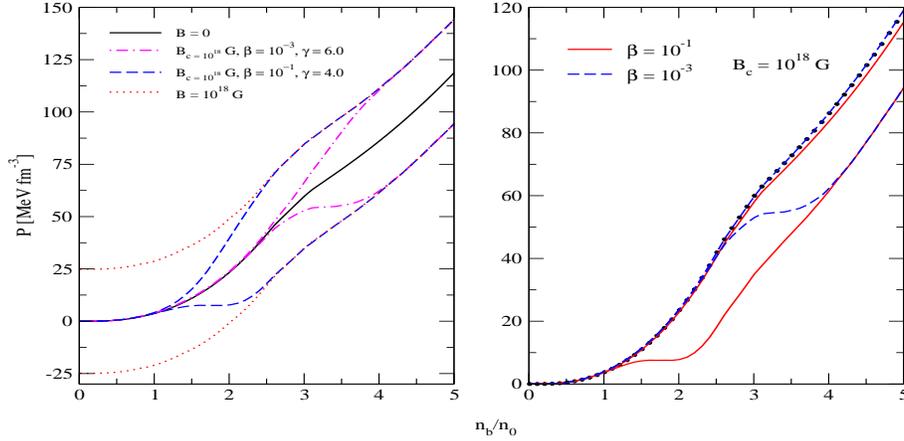


Fig. 1. Left panel (a): Variation of total pressure as a function of normalized baryon number density for fixed fields $B = 0$ (solid line), and $B_c = 10^{18}$ G and several density profiles: $\beta = 10^{-3}$, $\gamma = 6$ (dot-dashed lines), $\beta = 10^{-1}$, $\gamma = 4$ (dashed lines), and $\beta \rightarrow \infty$, i.e, $B = \text{constant}$ (dotted lines). For each pair of curves the upper branch is for P_{\perp} and the lower branch for P_{\parallel} . Right panel (b): Variation of total pressure as a function of normalized baryon number density for different magnetic field profiles and $B_c = 10^{18}$ G. The dots show the reference case $B = 0$. The solid and dashed lines correspond to $\beta = 0.1$ and 0.001 respectively. For each β we choose a pair of γ s; in the first case we have $\gamma = 1$ (upper), 4 (lower), whereas in the second case $\gamma = 1$ (upper), 6 (lower).

contribution is added to P , and consequently at a certain n_b , P ceases to increase and then decreases with the increase of n_b , rendering instability.

4. Conclusion

We have found that for sufficiently large magnetic fields with $B_c \sim 10^{18}$ G, the magnetar matter becomes unstable. The instability is associated with the anisotropic effects arised due to the magnetic field. The onset of instability depends on the magnetic field profile and B_c , which puts a natural upper bound for the central magnetic field of neutron stars, which is 5×10^{18} G.

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