# Gravitational energy of a noncommutative Vaidya black hole

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Abstract

In this paper we evaluate the components of the energy-momentum pseudotensors of Landau and Lifshitz for the noncommutative Vaidya spacetime. The effective gravitational mass experienced by a neutral test particle present at any finite distance in the gravitational field of the noncommutative Vaidya black hole is derived. Using the effective mass parameter one finds that the naked singularity is massless and this supports Seifert's conjecture.

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## I. INTRODUCTION

There are several models of noncommutative geometry leading to an effective grainy treatment of the spacetime manifold such that coordinate operators might fail to commute [1]. In 2005, Nicolini *et al* [2] in a physically inspired type of noncommutativity based on the coordinate coherent state method, via a minimal length induced by averaging noncommutative coordinate fluctuations [3], showed that the short distance behavior of point-like structures can be cured. They derived the metrics for some noncommutative black holes and found a new thermodynamically stable final stage of the Hawking radiation in which the curvature singularity of the black hole is removed [2, 4]. Moreover, their method is consistent with Lorentz invariance, unitarity and UV-finiteness of quantum field theory. On the other hand, the mass scale associated with noncommutativity, possibly and most reasonably is of the order of the Planck scale. This is supported by the fact that the fundamental Planck scale in models with large extra dimensions becomes as small as a TeV in order to solve the hierarchy problem. In addition, most of the phenomenological studies of the noncommutative models have assumed that the mass scale associated with noncommutativity cannot lie far above the TeV scale [5]. Therefore the hope is that if the noncommutativity scale is in the TeV range one might be able to detect some evidences for it at future particle colliders [6].

Recently, we investigated some aspects of the black hole thermodynamics by using the noncommutative geometry inspired formalism (the coordinate coherent state method) [7]. In this formalism the point-like particle, instead of being totally localized at a point via a Dirac-delta function distribution, is described as a smeared-like particle via a Gaussian distribution of minimal width  $\sqrt{\sigma}$ , where  $\sigma$  is the smallest fundamental unit of an observable area in the noncommutative coordinates, beyond which coordinate resolution is vague. As a matter of fact, because of the appearance of high energies at small scales of a noncommutative manifold, the influences of manifold quantum fluctuations turn out to be apparent and forbid any measurements to observe a particle location with a precision more than an intrinsic length scale. To clarify more features on these topics, see [8] and the references included.

In the context of black hole physics, a non-static and spherically symmetric spacetime is dependent upon an arbitrary dynamical mass function. In this manner, an important issue is raised in connection to how black hole mass decreases as a back-reaction of the Hawking radiation, and thus it may be appropriately presented by the Vaidya solution [9, 10]. The Vaidya black hole is considered as a kind of a more applicable one owing to its time-dependent decreasing mass. In this paper, we use the noncommutative Vaidya (NCV) metric derived in Ref. [11] to find its gravitational energy. There are several prescriptions for computing the energy for a non-static general relativistic system [12–19]. For instance, one can use Landau and Lifshitz's (LL) definition of energy [15], which is easier to work out. Since a neutral test particle situated at any finite radial distance in the gravitational field of the NCV black hole experiences an effective gravitational mass, so this is the subject of interest to us to evaluate the effective mass in this field.

The paper is organized in the following. In Sec. II, we briefly discuss concerning the energy-momentum localization in the literature. The energy-momentum distributions associated with the NCV black hole are evaluated in Sec. III and the LL definition of energy is employed for these computations. The summary is presented in Sec. IV. In this work, Greek (Latin) indices run from 0 to 3 (1 to 3) and we use the natural units with the following definitions:  $\hbar = c = G = 1$ .

### **II. ENERGY-MOMENTUM LOCALIZATION**

The energy-momentum localization is one of the most significant questions which survives uncertain in the theory of Einstein general relativity (GR). There are plenty of attempts to attain a specified approval for the explanation of energy-momentum as an important conserved quantity in GR. Unfortunately, there is still no generally confirmed explanation of energy and momentum distributions in the literature [20]. The energy-momentum conservation in the context of GR is given by

$$\nabla_{\alpha}T^{\alpha}_{\beta} = 0, \tag{1}$$

where  $T^{\alpha}_{\beta}$  shows the symmetric energy-momentum tensor containing the matter and all non-gravitational fields. An expression for energy-momentum complexes due to including the contribution from the gravitational field energy has been found by Einstein [12]. This expression is not a tensor and is defined as the gravitational field pseudotensor  $t^{\alpha}_{\beta}$ . The energy-momentum complex obeys the local conservation laws as follows:

$$\mathcal{T}^{\alpha}_{\beta,\alpha} \equiv \frac{\partial}{\partial x^{\alpha}} \left( \sqrt{-g} (T^{\alpha}_{\beta} + t^{\alpha}_{\beta}) \right) = 0, \tag{2}$$

where g is the determinant of the metric tensor  $g_{\alpha\beta}$ . The energy-momentum complex  $\mathcal{T}^{\alpha}_{\beta}$  is substituted for the energy-momentum tensor  $T^{\alpha}_{\beta}$  which is a mixture of  $T^{\alpha}_{\beta}$  and  $t^{\alpha}_{\beta}$  but in the usual form of conservation laws. Then, we can write

$$\mathcal{T}^{\alpha}_{\beta} = \theta^{\alpha\gamma}_{\beta,\gamma},\tag{3}$$

where  $\theta_{\beta}^{\alpha\gamma}$  are marked like superpotentials which are not uniquely defined. With a suitable choice of coordinates, one can vanish the pseudotensor  $t^{\alpha}_{\beta}$  at a special point, e.g. Schrödinger showed that the pseudotensor can be disappeared outside the schwarzschild radius. Numerous works have been accomplished by using a more appropriate quantity to exhibit the energy and momentum distributions of various gravitational backgrounds. The different descriptions for the energy-momentum complexes [12–19] can only provide important results if the calculations are performed in Cartesian coordinates. Penrose 21 introduced a method of quasi-local energy to obtain the energy-momentum of a curved spacetime via applying any coordinate system. Many research workers [22] have regarded a class of various suggestions of the quasi-local energy to investigate various models of the universe. Nevertheless the formalism of energy-momentum complexes could not produce some unique explanation of energy in the framework of GR due to the fact that each of these quasi-local expressions have their own topics. In 1996, it was shown that the deferent energy-momentum complexes coincide for any Kerr-Schild class metric and they give the same energy distribution [23] in which their outcomes are similar to the results of Penrose [21] and also Tod [24] utilizing the notion of quasi-local mass. Many universal outcomes of the most general non-static spherically symmetric spacetime by utilizing the Kerr-Schild class metric was found afterwards by Virbhadra [25]. In 1999, Chang *et al* [26] demonstrated that every energy-momentum complex is connected to a Hamiltonian boundary term. Hence, the energy-momentum complexes are quasi-local and suitable. It would be worthwhile to denote that, in the next section we carry out the calculations in quasi-Cartesian coordinates due to the fact that the energy associated with the NCV metric using the LL energy-momentum complex provides a meaningful result if quasi-Cartesian coordinates are chosen (for instance, see [27]).

### **III. ENERGY-MOMENTUM DISTRIBUTIONS IN NCV SPACETIME**

Since the dynamics for the black hole mass is a significant problem, we therefore focus on a NCV spacetime which exhibits a particularly rich dynamical structure. According to the noncommutative geometry inspired method, one can obtain the NCV metric in the presence of a smeared mass source by solving Einstein equations [11]. Let us first consider the diagonal form of the NCV metric with respect to  $\{x^{\alpha}\} = \{t, r, \theta, \phi\}$  coordinates as follows:

$$ds^{2} = \left(1 - \frac{2\mathcal{M}_{\sigma}}{r}\right)dt^{2} - \left(1 - \frac{2\mathcal{M}_{\sigma}}{r}\right)^{-1}dr^{2} - r^{2}d\Omega^{2},\tag{4}$$

where  $d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2$  is the line element on the 2-dimensional unit sphere and  $\mathcal{M}_{\sigma}$  is the gaussian-smeared mass distribution given by

$$\mathcal{M}_{\sigma} = M_{I} \left[ \mathcal{E} \left( \frac{r-t}{2\sqrt{\sigma}} \right) \left( 1 + \frac{t^{2}}{2\sigma} \right) - \frac{r}{\sqrt{\pi\sigma}} e^{-\frac{(r-t)^{2}}{4\sigma}} \left( 1 + \frac{t}{r} \right) \right], \tag{5}$$

where  $M_I$  is the initial black hole mass and  $\mathcal{E}(n)$  is the Gauss error function defined as  $\mathcal{E}(n) \equiv 2/\sqrt{\pi} \int_0^n e^{-p^2} dp$ . In the static case, t = 0, one regains the same result as Ref. [2], i.e. the noncommutative Schwarzschild metric and in the limit  $\sigma \to 0$  with t = 0, one recovers the ordinary Schwarzschild solution. We require to reexpress the NCV metric in quasi-Cartesian coordinates. Transforming (4) to Cartesian terms according to  $x = r \sin \theta \cos \phi$ ,  $y = r \sin \theta \sin \phi$ , and  $z = r \cos \theta$ , one gets the metric

$$ds^{2} = \left(1 - \frac{2\mathcal{M}_{\sigma}}{r}\right)dt^{2} - dx^{2} - dy^{2} - dz^{2} - \frac{2\mathcal{M}_{\sigma}}{r^{3} - 2\mathcal{M}_{\sigma}r^{2}}\left(xdx + ydy + zdz\right)^{2}, \quad (6)$$

where  $r^2 = x^2 + y^2 + z^2$ . Now we are ready to express Einstein's equations as defined by LL [15]

$$L^{\alpha\beta} = \frac{1}{16\pi} \mathcal{S}^{\alpha\beta\gamma\delta}_{,\gamma\delta},\tag{7}$$

where  $L^{\alpha\beta}$  is the energy-momentum complex and incorporates both the stress-energy tensor of matter and an expression quadratic in first derivatives of the metric which is symmetric with respect to its indices. The right-hand side, namely LL superpotentials  $S^{\alpha\beta\gamma\delta}$ , includes an expression in which it has symmetries similar to the curvature tensor and can be written as

$$S^{\alpha\beta\gamma\delta} = -g \left( g^{\alpha\beta} g^{\gamma\delta} - g^{\alpha\gamma} g^{\beta\delta} \right).$$
(8)

The LL energy-momentum complex satisfies the local conservation laws

$$L^{\alpha\beta}_{\ ,\beta} = 0. \tag{9}$$

The  $L^{00}$  is the energy density and  $L^{0i}$  are the momentum density components. The energy and momentum in the LL prescription for a four-dimensional background are given by

$$P^{\alpha} = \int \int \int L^{0\alpha} dx^1 dx^2 dx^3.$$
<sup>(10)</sup>

These integrals are extended over all space for t = const, and have to be limited to the utilize of quasi-Cartesian coordinates to have the meaning of energy and momentum. In order to evaluate the energy and momentum distributions, we firstly have to compute the LL superpotentials. There are seventy two nonzero superpotentials in the LL prescription for the NCV spacetime but the required ones are the following

$$S^{1010} = -S^{0011} = \frac{r^3 - 2M_{\sigma}x^2}{r^3 - 2M_{\sigma}r^2},$$

$$S^{2010} = S^{1020} = -S^{0021} = -S^{0012} = -\frac{xyM_{\sigma}}{r^3 - 2M_{\sigma}r^2},$$

$$S^{3010} = S^{1030} = -S^{0031} = -S^{0013} = -\frac{xzM_{\sigma}}{r^3 - 2M_{\sigma}r^2},$$

$$S^{2020} = -S^{0022} = \frac{r^3 - 2M_{\sigma}y^2}{r^3 - 2M_{\sigma}r^2},$$

$$S^{3020} = S^{2030} = -S^{0032} = -S^{0023} = -\frac{yzM_{\sigma}}{r^3 - 2M_{\sigma}r^2},$$

$$S^{3030} = -S^{0033} = \frac{r^3 - 2M_{\sigma}z^2}{r^3 - 2M_{\sigma}r^2}.$$
(11)

Substituting LL superpotentials into Eq. (7), the energy and momentum densities take the form

$$L^{00} = \frac{1}{4\pi (r-2\mathcal{M}_{\sigma})^2} \left[ \frac{\mathcal{M}_{\sigma}}{r} \left( \frac{9(r-2\mathcal{M}_{\sigma})^2}{r^2} - 1 \right) + \mathcal{M}'_{\sigma} \right],$$

$$L^{01} = -\frac{2x\dot{\mathcal{M}}_{\sigma}}{8\pi r (r-2\mathcal{M}_{\sigma})^2},$$

$$L^{02} = -\frac{2y\dot{\mathcal{M}}_{\sigma}}{8\pi r (r-2\mathcal{M}_{\sigma})^2},$$

$$L^{03} = -\frac{2z\dot{\mathcal{M}}_{\sigma}}{8\pi r (r-2\mathcal{M}_{\sigma})^2}.$$
(12)

The prime abbreviates  $\partial/\partial r$ , and the overdot abbreviates  $\partial/\partial t$ . The quantities  $\mathcal{M}'_{\sigma}$  and  $\dot{\mathcal{M}}_{\sigma}$  are given by

$$\mathcal{M}'_{\sigma} = \frac{M_I r^2}{2\sqrt{\pi\sigma^3}} e^{-\frac{(r-t)^2}{4\sigma}},\tag{13}$$

$$\dot{\mathcal{M}}_{\sigma} = \frac{M_{I}}{\sqrt{\pi\sigma}} \left[ \sqrt{\frac{\pi}{\sigma}} t \, \mathcal{E}\left(\frac{r-t}{2\sqrt{\sigma}}\right) - \left(\frac{r^{2}}{2\sigma} + 2\right) e^{-\frac{(r-t)^{2}}{4\sigma}} \right]. \tag{14}$$

We are interested in computing the energy-momentum distributions associated with the NCV black hole background, which are contained in a sphere of radius  $r_0$ . Therefore, if we replace Eqs. (12) into Eq. (10), applying the Gauss theorem and evaluating the integrals over the surface of two-sphere of radius  $r_0$ , we get the energy and momentum components in the following form

$$E(r_0) = M_I - \frac{\mathcal{M}_\sigma(r_0)r_0}{r_0 - 2\mathcal{M}_\sigma(r_0)},\tag{15}$$

$$P^1 = P^2 = P^3 = 0. (16)$$

Note that the asymptotic value of the total gravitational mass of a NCV black hole is the initial black hole mass  $M_I$ . The energy distribution derived here is indeed the energy of the gravitational field that a neutral particle experiences at a finite distance  $r_0$ . Hence, the energy given by Eq. (15) can also be identified as the effective gravitational mass  $M_{eff}$  of the spacetime under consideration. Many authors have devoted a worthwhile attention to the problem of finding the effective mass for various spacetimes [28]. Studying on this issue was first considered by Cohen and Gautreau [29]. They were the first who introduced the idea of effective mass by utilizing Whittaker's theorem and derived the effective mass for the Reissner-Nordström and Kerr-Newman spacetimes.

In addition, the result in Eq. (15) may be worthwhile for testing Seifert's conjecture in the context of the naked singularity arising from the spherical collapse described by the metric (4). In 1979, Seifert [30] conjectured that any singularity which appears is invisible if a finite nonzero mass collapses into a point, or is naked if either one has singularities along lines (or surfaces) or the central singularities are carefully arranged that they include only zero mass. In Ref. [11], we analyzed the metric (4) in three possible causal structures and found a zero remnant mass at long times, i.e. an instable black hole remnant. So, a naked singularity at r = 0 in this non-static case is appeared which is clear from Eq. (15) that at the origin, the effective mass of the NCV black hole is equal to zero. This supports the Seifert conjecture. The naked singularity forming in the Vaidya null dust collapse is also massless [25], providing a support to Seifert's hypothesis which approves our outcome.

Here, it is important to stress that the energy density given by the first of the expressions in Eq. (12) becomes infinite when we set r equal to the horizon radius  $r_H$ , so that  $r_H = 2\mathcal{M}_{\sigma}(r_H)$ . As a result, the energy distribution becomes infinite for the case of the black hole horizon. However, the condition of the infiniteness for the energy density of a closed system which is confined to a two-sphere of radius  $r_0$  is not a sensible condition, thereby, this value for the radial coordinate should be eliminated. On the other hand, when one considers the time-varying mass in the NCV black hole, one could possibly deal with this singularity at long times. According to Ref. [11], as time moves forward the minimal nonzero mass decreases but the minimal nonzero horizon radius increases which means that in the long-time limit, the horizon radius tends to the infinity.

## IV. SUMMARY

In summary, we have evaluated the energy and momentum distributions in the NCV black hole background using the LL prescription. The energy distribution of the NCV black hole that is contained in a sphere of radius  $r_0$  shows that a neutral test particle situated at a finite distance  $r_0$  experiences the gravitational field of the effective gravitational mass in the spacetime under study, while the momentum distributions  $P^i$  are equal to zero which are acceptable results. Using the expression for energy we have found that the naked singularity forming in the NCV black hole is massless and this confirms the Seifert conjecture.

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