

Supersymmetry of classical solutions in Chern-Simons higher spin supergravity

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ABSTRACT: We construct and study classical solutions in Chern-Simons supergravity based on the superalgebra $sl(N|N-1)$. The algebra for the $N=3$ case is written down explicitly using the fact that it arises as the global part of the super conformal \mathcal{W}_3 superalgebra. For this case we construct new classical solutions and study their supersymmetry. Using the algebra we write down the Killing spinor equations and explicitly construct the Killing spinor for conical defects and black holes in this theory. We show that for the general $sl(N|N-1)$ theory the condition for the periodicity of the Killing spinor can be written in terms of the products of the odd roots of the super algebra and the eigenvalues of the holonomy matrix of the background. Thus the supersymmetry of a given background can be stated in terms of gauge invariant and well defined physical observables of the Chern-Simons theory. We then show that for $N \geq 4$, the $sl(N|N-1)$ theory admits smooth supersymmetric conical defects.

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1. Introduction

Consistent theories of interacting higher spin fields constructed by Vasiliev [1] have been the focus of many recent works. For a review of higher spin theories see [2]. These theories are interesting from the perspective of the AdS/CFT since they are examples of gravitational backgrounds in which one does not need to deal with the entire spectrum of massive string excitations but only with infinite set of higher spin fields. Higher spin theories on AdS_4 have been proposed as dual descriptions of vector like field theories [3, 4, 5], sub-sectors of free Yang-Mills theories [6, 7, 8, 9], and very recently argued to be duals of certain ABJ models [10].

Higher spin theories in three space time dimensions are particularly tractable since in this situation the Vasiliev like theories can be formulated in terms of a Chern-Simons theory [11]. Furthermore in three dimensions, it is not necessary to consider an infinite number of higher spin fields to obtain consistent interactions. It is possible to work with a finite set of higher spin fields. Vasiliev like theories in 3

dimensions coupled to a massive complex scalar have been proposed to be holographic duals to \mathcal{W}_N minimal models based on the coset [12]

$$\frac{SU(N)_k \otimes SU(N)_1}{SU(N)_{k+1}}. \quad (1.1)$$

This duality is a new example for the AdS_3/CFT_2 correspondence and various checks of the proposal include matching of the symmetries, comparison of the one loop partition function and the three point correlators. For a comprehensive list of references please see [13]. A supersymmetric extension of the higher spin/minimal model duality has been proposed in [14]. This duality has also been checked by comparison of the symmetries and the partition function [15, 16]. Chern-Simons theories based on super-extended higher spin super algebras have been considered in [17] and their asymptotic algebras have been shown to agree with the corresponding super conformal \mathcal{W}_∞ algebra.

Studying classical solutions in Chern-Simons theories based on a higher spin group provides more more insights to holography in three dimensions. The higher spin black holes found in [18] and the conical defect solutions [19] have proved to be useful to study aspects of the holographic renormalization group and the nature of singularities in higher spin gravity [20, 21]. In fact the higher spin black hole studied in [18] is dual to a renormalization group flow between two CFT's. Smooth conical defects have been argued to be dual to the primaries in the \mathcal{W}_N minimal model after an appropriate analytic continuation [19].

Motivated by these developments in bosonic higher spin theories we study and construct new classical solutions in Chern-Simons theories based on the $sl(N|N-1)$ super algebra. Any classical solution of the bosonic theory can be embedded as a solution in the supersymmetric theory. In addition to these solutions, supersymmetric theories also admit solutions in which additional fields required for the supersymmetric completion are turned on. In this paper we find such solutions in the Chern-Simons theory based on $sl(3|2)$ super algebra. These solutions are conical defects and black holes, which have fields valued in $sl(2)$ and the $u(1)$ part of the connection in addition to the $sl(3)$. The main motivation to construct the solutions in this paper is to study their supersymmetry. The study of supersymmetry in higher spin theories is a new subject and as far as we are aware there are no general results for when a classical solution is supersymmetric in higher spin theories. An early study of supersymmetry of a black hole solution in a higher spin theory on AdS_4 is [22]. The Killing spinor equations for this case are quite involved and difficult to solve. We will see that Killing spinors for solutions in supersymmetric higher spin Chern-Simons theories are considerably easier to obtain. In fact we will obtain a general condition for when a classical solution is supersymmetric. This condition can be stated invariantly in terms of the eigenvalues of the holonomy of the classical solution. This is important since the Chern-Simons action is independent of

the metric on the manifold and the eigenvalues of the holonomy are the only gauge independent well defined physical observables.

Our working example will be the algebra $sl(3|2)$ which is the global part of the \mathcal{W}_3 super algebra in the large central charge limit. All these theories have two $U(1)$ gauge fields corresponding to the R symmetry of the dual conformal field theory. The supersymmetric conditions in the Chern-Simons theory based on supergroups which contained spins ≤ 2 were earlier analyzed in [23, 24, 25, 26, 27]. Once the background flat connection is given, the Killing spinor equations are particularly easy to write for a Chern-Simons theory based on any supergroup. The Killing spinor equation is just a covariant derivative with the flat connection as the background. Thus the supergroup structure is sufficient to write down the Killing spinor equation. By studying various solutions we arrive at the observation that the solution admits a periodic Killing spinor if the combined $U(1)$ part of the holonomy together with the holonomy of the rest of the connection around the angular direction in AdS_3 is trivial. This observation enables us to state the condition on the periodicity of the Killing spinor in terms of the odd roots of the $sl(N|N - 1)$ super algebra and the eigenvalues of the holonomy matrix. This is one of the key results of this work. The reader can directly turn to section 4.1 and refer to equation (4.9) for this result. Using this condition for the periodicity of the Killing spinor we show that for $N \geq 4$ the $sl(N|N - 1)$ theory admits smooth supersymmetric conical defects. These solutions should play a crucial role in obtaining the duals of the chiral primaries in the supersymmetric minimal \mathcal{W}_N model proposed in [14].

The organization of this paper is as follows: In the next section we review some generalities of higher spin AdS_3 supergravity and write down the Killing spinor equation for any Chern-Simons theory based on a given super group. We then provide the details of the commutation relations for the $sl(3|2)$ super algebra. We derive them by considering the global part and the large central charge limit of the super \mathcal{W}_3 conformal algebra written down in [28]. In section 3 we study the supersymmetry of various classical solutions in the Chern-Simons theory based on the $sl(3|2)$ super algebra. These include the BTZ black hole, the black hole with higher spin field. They also include a new black hole solution, this background has fields valued in the $sl(2)$ required for the supersymmetric completion of the bosonic $sl(3)$ turned on. We then study the supersymmetry of conical defects in these theories. Again these defects also include those with fields in the $sl(2)$ turned on. A summary of the supersymmetric conditions for these backgrounds is provided in table 1 of section 3.5. In section 4 we show that the periodicity requirement of the Killing spinor in the angular direction can be cast in terms of the holonomies of the background flat connection. We show that the supersymmetric conditions of any background can be written in terms of products of the odd roots of the super algebra with eigenvalues of the holonomy matrix of the background. We then use this result to show that for $N \geq 4$, the $sl(N|N - 1)$ theory admits smooth supersymmetric conical defects.

Section 5 contains the conclusions and a discussion of the results.

Note added: After completion of this work, we received [29] which overlaps with some portions of this paper.

2. Chern-Simons higher spin supergravity

It is well known that pure gravity in AdS_3 can be written in terms of difference of two Chern-Simons actions based on the algebra $sl(2, R)$ [30]. Similarly supersymmetric extensions of pure gravity containing spins ≤ 2 can be written as a Chern-Simons action based on supersymmetric extensions of $sl(2, R)$ [31]. Since higher spin theories containing only bosonic fields are based on the the $sl(N, R)$ with $N > 2$ [32, 33], it is natural to look for supersymmetric extensions of the $sl(N, R)$ algebra to construct consistent interacting higher spin theories in AdS_3 containing fermions. Given any such super algebra \mathcal{G} the parity invariant Chern-Simons action is given by

$$S = \frac{k}{2\pi} \int \left[\text{str} \left(\Gamma d\Gamma + \frac{2}{3} \Gamma^3 \right) - \text{str} \left(\tilde{\Gamma} d\tilde{\Gamma} + \frac{2}{3} \tilde{\Gamma}^3 \right) \right]. \quad (2.1)$$

Here $\Gamma, \tilde{\Gamma}$ are the 1-forms which take values in \mathcal{G} and str refers to the super-trace over the respective algebras. The integral is over the 3 dimension space time. The equations of motion of this action are the following flatness conditions

$$d\Gamma + \Gamma \wedge \Gamma = 0, \quad d\tilde{\Gamma} + \tilde{\Gamma} \wedge \tilde{\Gamma} = 0. \quad (2.2)$$

To obtain the equations of motion in component form, one needs to expand $\Gamma, \tilde{\Gamma}$ in terms of the generators of the super algebra. The coefficients of this expansion are the fields of the theory, this is then substituted in the equations given in (2.2) to obtain the equations of motion in the component form. Thus to write down the equations of motion it is sufficient to know the structure constants of the algebra.

The generalized Killing spinor equations

It is also easy to write down the Killing spinor equations. Let the bosonic generators of the algebra be denoted by T_a and the corresponding bosonic fields by A^a . Similarly let the fermionic generators be G_i . Consider a bosonic solution to the equations of motion. Then one has the following equation

$$d(A^a T_a) + (A^a T_a) \wedge (A^b T_b) = 0, \quad (2.3)$$

where the bosonic fields A^a are 1-forms. The Killing spinor equation is essentially the equation that demands that the background A^a is invariant under fermionic gauge transformations. Let ϵ^i be the parameters of this transformation, then the equation for the Killing spinor is given by

$$\delta\psi \equiv \partial_\mu \epsilon^i G_i + A_\mu^a \epsilon^i [T_a, G_i] = 0. \quad (2.4)$$

This is essentially the equation demanding that the covariant derivative in presence of the bosonic background A_μ^a vanishes. The solutions ϵ^i are the Killing spinors. It is clear that the variation $\delta\psi$ is a fermionic symmetry of the Lagrangian since it is just a gauge transformation. By demanding $\delta\psi = 0$ we are looking for general variations with parameters involving fermions which leaves the background invariant. In general the fermionic fields ϵ^i can contain fermions with spins $s \geq 1/2$. This is the generalized notion of the Killing spinor in the higher spin theory. It is important to note that flatness conditions in (2.3) are the integrability constraints of the Killing spinor equation (2.4). Thus given that a bosonic background satisfies the equations of motion, solutions to the Killing spinor equations are guaranteed to exist. However we must also impose the condition that the Killing spinors are periodic with respect to the angular co-ordinate in AdS_3 . This then decides the condition whether a given background is supersymmetric.

The class of super algebras we will be interested in belongs to $sl(N|N-1)$. We will also examine the supersymmetry in one copy of the $sl(N|N-1) \oplus sl(N|N-1)$ Chern-Simons theory. However the central conclusion regarding the supersymmetry of a given background in terms of the eigenvalues of the holonomy of the background drawn at the end our analysis applies to any super algebra. An appropriate basis to discuss the $sl(N|N-1)$ algebra is the explicit matrix representation of the algebra given in section 61 of [34]. This is in the Cartan-Weyl basis which is suitable for the general analysis of the Killing spinor and the supersymmetric conditions. We will explicitly study the case of $sl(3|2)$. The bosonic part of this algebra is given by $sl(3) \oplus sl(2) \oplus u(1)$. This algebra contains the super group $sl(2|1)$ on which (2, 2) supergravity in AdS_3 is based.

2.1 The $sl(3|2)$ superalgebra

In this section we write down the commutation relations of $sl(3|2)$. We obtain this by taking the large central charge and the global part of the $\mathcal{N} = 2$ super \mathcal{W}_3 algebra written down by [28]. This provides evidence evidence that the boundary theory of Chern-Simons gravity based on $sl(3|2)$ is a super conformal theory with $\mathcal{N} = 2$ super \mathcal{W}_3 symmetry.

$\mathcal{N} = 2$ super \mathcal{W}_3 algebra contains generators with J, G^\pm, L with spin 1, 3/2 and 2 respectively. These generators obey the $\mathcal{N} = (2, 2)$ super conformal algebra among themselves. J is the generator of the R-symmetry, G^\pm are the supersymmetry generators and L is the stress tensor. In addition to this there is also the generators V, U, W with spin 2, 5/2, 3 respectively. W generates the super conformal \mathcal{W}_3 symmetry. Taking the large central charge limit and the global part of the commutation relations of $\mathcal{N} = 2$ super conformal \mathcal{W}_3 we obtain the following algebra for the

bosonic generators:

$$\begin{aligned}
[J, J] &= 0, & [L_m, L_n] &= (m-n)L_{m+n}, \\
[V_m, V_n] &= (m-n)(L_{m+n} + \kappa V_{m+n}), \\
[W_m, W_n] &= \frac{1}{4}(m-n)(2m^2 + 2n^2 - mn - 8)(L_{m+n} + \frac{\kappa}{5}V_{m+n}), \\
[J, L_n] &= 0, & [J, V_n] &= 0, & [J, W_n] &= 0, \\
[L_m, V_n] &= (m-n)V_{m+n}, & [L_m, W_n] &= (2m-n)W_{m+n}, \\
[V_m, W_n] &= \frac{\kappa}{5}(2m-n)W_{m+n}.
\end{aligned} \tag{2.5}$$

Here the subscripts m, n on the generators L run from $-1, 0, 1$ while the subscripts on the generators W run from $-2, -1, 0, 1, 2$. The commutation relations between bosonic and fermionic generators are given by

$$\begin{aligned}
[L_m, G_r^\pm] &= (\frac{1}{2}m - r)G_{m+r}^\pm, & [J, G_r^\pm] &= \pm G_r^\pm, \\
[L_m, U_r^\pm] &= (\frac{3}{2}m - r)U_{m+r}^\pm, & [J, U_r^\pm] &= \pm U_r^\pm, \\
[V_m, G_r^\pm] &= \pm U_{r+m}^\pm, & [G_r^\pm, W_m] &= (2r - \frac{1}{2}m)U_{r+m}^\pm, \\
[V_m, U_r^+] &= \frac{2}{5}\kappa(\frac{3}{2}m - r)U_{m+r}^+ + \frac{1}{4}(3m^2 - 2mr + r^2 - \frac{9}{4})G_{m+r}^+, \\
[V_m, U_r^-] &= -\frac{2}{5}\kappa^*(\frac{3}{2}m - r)U_{m+r}^- - \frac{1}{4}(3m^2 - 2mr + r^2 - \frac{9}{4})G_{m+r}^-, \\
[U_r^+, W_m] &= \frac{\kappa}{10}(2r^2 - 2rm + m^2 - \frac{5}{2})U_{r+m}^+ \\
&\quad + \frac{1}{8}(4r^3 - 3r^2m + 2rm^2 - m^3 - 9r + \frac{19}{4}m)G_{r+m}^+, \\
[U_r^-, W_m] &= \frac{\kappa^*}{10}(2r^2 - 2rm + m^2 - \frac{5}{2})U_{r+m}^- \\
&\quad + \frac{1}{8}(4r^3 - 3r^2m + 2rm^2 - m^3 - 9r + \frac{19}{4}m)G_{r+m}^-.
\end{aligned} \tag{2.6}$$

Here the subscripts r, s on G^\pm run from $-1/2, 1/2$ while the subscripts on the generators U^\pm run from $-3/2, -1/2, 1/2, 3/2$. Finally the anti-commutation rules between the fermionic generators are given by

$$\begin{aligned}
\{G_r^\pm, G_s^\mp\} &= 2L_{r+s} \pm (r-s)J, & \{G_r^\pm, G_s^\pm\} &= 0, \\
\{G_r^\pm, U_s^\mp\} &= 2W_{r+s} \pm (3r-s)V_{r+s}, & \{G_r^\pm, U_s^\pm\} &= 0, \\
\{U_r^+, U_s^-\} &= -\frac{2}{5}\kappa(r-s)W_{r+s} + (3s^2 - 4rs + 3r^2 - \frac{9}{2})(\frac{1}{2}L_{r+s} + \frac{\kappa}{5}V_{r+s}) \\
&\quad + \frac{1}{4}(r-s)(r^2 + s^2 - \frac{5}{2})J_{r+s}, \\
\{U_r^\pm, U_s^\pm\} &= 0.
\end{aligned} \tag{2.7}$$

On taking large central charge limit, the non-linear terms in the super \mathcal{W}_3 algebra drop off and we obtain $\kappa = \pm(5/2)i$. We have verified that all the Jacobi identities of this algebra are satisfied using the Quantum add-on for Mathematica [35].

To see that the bosonic part of the algebra given in (2.5) is given by the direct sum $sl(3) \oplus sl(2) \oplus u(1)$, we define the following linear combinations of generators

$$T_m^+ = -\frac{1}{3}(L_m + 2iV_m) \quad T_m^- = \frac{1}{3}(4L_m + 2iV_m). \tag{2.8}$$

Substituting these redefinitions in (2.5) we obtain

$$[T_m^+, T_n^-] = 0, \quad [T_m^+, W_n] = 0, \quad (2.9)$$

and we can show that the generators T_m^+ obey the $sl(2)$ algebra while the generators T_n^-, W_m obey the commutation relations of the $sl(3)$ algebra given by

$$\begin{aligned} [T_m^-, T_n^-] &= (m-n)T_{m+n}^-, & [T_m^-, W_n] &= (2m-n)W_{m+n}, \\ [W_m, W_n] &= \frac{3}{16}(m-n)(2m^2 + 2n^2 - mn - 8)T_{m+n}^-. \end{aligned} \quad (2.10)$$

Note that comparing the $sl(3)$ algebra given in equation (A.2) of [18] we see that the parameter σ defined in those equations is equal to $(3/4)^2$. Now that we have the explicit $sl(3|2)$ algebra we can proceed to obtain solutions to the equations of motion and study their supersymmetry. The traces of the product of any two of the $sl(3)$ generators is the same as that of equation (A.3) of [18] with $\sigma = (3/4)^2$, while for the $sl(2)$ we use the representation in terms of the Pauli matrices.

3. Supersymmetry of classical solutions

We begin this section by describing the general strategy we adopt to find the Killing spinors for the various backgrounds considered in this paper. We reduce the Killing spinor equation to a set of ordinary first order equations with constant coefficients which can then be easily solved. In section 3.2 we construct the general higher spin conical defects in the $sl(3|2)$ theory. These solutions in general have fields in the $sl(3) \oplus sl(2) \oplus u(1)$ directions. We then solve the Killing spinor equations and determine the supersymmetric conditions for the supercharges with $u(1)$ charge in one copy of $sl(3|2)$ in the Chern-Simons theory. This analysis can be generalized for the remaining charges. We also determine the special values in the parameter space of conical defects which reduce to AdS_3 . In section 3.3 we study the supersymmetry of black holes in this theory. This includes the usual BTZ black hole embedded in $sl(2)$, the higher spin black hole of [18] embedded in $sl(3)$ along with the $u(1)$ turned on. We also construct a new black hole solution which has charges in $sl(3) \oplus sl(2) \oplus u(1)$ and study its supersymmetry. The list of all the solutions studied and the corresponding supersymmetry conditions is given in table 1.

3.1 General strategy to obtain the Killing spinors

The gauge connections in the $sl(3|2)$ theory which will be of interest in this paper has the following generic form

$$\begin{aligned} A = \left(\sum_{m=-1}^1 (t_m e^{m\rho} T_m^- + s_m e^{m\rho} T_m^+) + \sum_{m=-2}^2 (w_m e^{m\rho} W_m) + \xi J \right) dx^+ \\ - \xi J dx^- + (T_0^+ + T_0^-) d\rho. \end{aligned} \quad (3.1)$$

Here $x^\pm = t \pm \phi$ and ρ, t, ϕ are the radial, time and the angular co-ordinates of the three dimensional spaces we consider. The connection in (3.1) obeys the flatness condition. The general form of the connection can be conveniently written as $A = a_\mu^n e^{(n)\rho} T_n$. T_n being a bosonic generator of the superalgebra. Negative weights of the generators with respect to $L_0 = tp_0 + T_0^-$ appear in the exponential factors. For example, we have terms like $w_+^{(2)} e^{2\rho} W_2$ and $t_+^{(-1)} e^{-\rho} T_{-1}^+$.

The equation for Killing spinor is given by

$$(\partial_\mu \epsilon^r) G_r + \epsilon^a A_\mu^b [T_b, G_a] = 0, \quad (3.2)$$

where $[T_b, G_a]$ is some linear combination of the fermionic generators which we can write as $f_{bac} G_c$. Here f_{bac} are the structure constants of the superalgebra and b is a bosonic index while a and c are fermionic ones. Substituting for the commutation relation in (3.2) we obtain the following equation

$$(\partial_\mu \epsilon^r) G_r + \epsilon^a A_\mu^b f_{bac} G_c = 0, \quad (3.3)$$

To write the above equation in matrix form we define the matrix $(\mathbb{M}_\mu)_{ac} = A_\mu^b f_{bac}$. Using this defining the killing spinor equation reduces to

$$\partial_\mu \epsilon^c + (\mathbb{M}_\mu)_a^c \epsilon^a = 0. \quad (3.4)$$

Our task now is to solve (3.4). In order to do this we make the following ansatz for the solution.

$$\epsilon = \mathcal{R}(\rho) e^{\xi x_-} f(x_+), \quad (3.5)$$

where $\mathcal{R}(\rho)$ is a square matrix which is given by

$$\mathcal{R}(\rho) = \begin{pmatrix} e^{-\rho/2} & 0 & 0 & 0 & 0 & 0 \\ 0 & e^{\rho/2} & 0 & 0 & 0 & 0 \\ 0 & 0 & e^{-3\rho/2} & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{-\rho/2} & 0 & 0 \\ 0 & 0 & 0 & 0 & e^{\rho/2} & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{3\rho/2} \end{pmatrix}. \quad (3.6)$$

This ansatz solves the ρ dependence because the matrix \mathbb{M}_ρ has the form $\text{Diag}(1/2, -1/2, 3/2, 1/2, -1/2, -3/2)$.

We now show that connections of the type (3.1) obey the following property :

$$\mathcal{R}^{-1}(\rho) \mathbb{M}_\mu \mathcal{R}(\rho) \text{ is independent of } \rho \quad (3.7)$$

This can be seen by considering the definitions of \mathbb{M}_μ and $\mathcal{R}(\rho)$. Substituting their definitions we obtain the following

$$\begin{aligned} \mathcal{R}_{ea}^{-1}(\mathbb{M}_\mu)_{ac} \mathcal{R}_{cd} &= (e^{-(a)\rho} \delta_{ea}) (f_{bac} A_\mu^b) (e^{(c)\rho} \delta_{cd}), \\ &= (e^{-(a)\rho} \delta_{ea}) (f_{bac} a_\mu^b e^{(b)\rho}) (e^{(c)\rho} \delta_{cd}), \\ &= e^{-(a+b-c)\rho} \delta_{ea} \delta_{cd} f_{bac} a_\mu^b. \end{aligned} \quad (3.8)$$

Note that the exponential factors are negative weights of the corresponding generators. Since $[T_b, G_a] \sim G_{a+b}$, f_{bac} is non-zero only when $a + b = c$. Thus the ρ dependence drops off from (3.8). This allows us to conclude that the connections obey the property given in (3.7). Finally we obtain

$$\mathcal{R}_{ea}^{-1}(\mathbb{M}_\mu)_{ac}\mathcal{R}_{cd} = f_{bed}a_\mu^b. \quad (3.9)$$

Now $f(x_+)$ in (3.5) is a column vector which solves the x_+ dependence of the Killing spinor. Using the property (3.7) in the $+$ component of the Killing spinor equation (3.4) we obtain

$$\partial_+ f(x_+) + [\mathcal{R}^{-1}(\mathbb{M}_+) \mathcal{R}] f(x_+) = 0. \quad (3.10)$$

Now let λ_i be the eigenvalues of the constant matrix $\mathcal{R}^{-1}(\mathbb{M}_+) \mathcal{R}$, then the solution for the above equation is

$$f(x_+) = \sum_i c_i e^{-\lambda_i x_+} \mathbf{z}_i, \quad (3.11)$$

where \mathbf{z}_i is the eigenvector of $\mathcal{R}^{-1}(\mathbb{M}_+) \mathcal{R}$ corresponding to the eigenvalue λ_i . Finally the x_- dependence of the Killing spinor is captured by the simple factor $e^{\xi x_-}$. This is due to the fact that $(\mathbb{M}_-)_{cd} = -\xi \delta_{cd}$.

3.2 Conical defects

Metric and gauge connections

We shall generalize the solution of [19] to include the spin-1 gauge field corresponding to the generators J and the additional spin-2 field corresponding to the generators V . We start with the 1-forms, written in terms of the decoupled generators, T^+ and T^- as defined in (2.8)

$$A = (e^{-\rho} \delta_{-1} T_{-1}^+ + e^\rho \delta_1 T_1^+ + e^{-\rho} \beta_{-1} T_{-1}^- + e^\rho \beta_1 T_1^- + e^{-\rho} \eta_{-1} W_{-1} + e^\rho \eta_1 W_1 + \xi J) dx^+ - \xi J dx^- + (T_0^- + T_0^+) d\rho, \quad (3.12)$$

$$\bar{A} = -(e^{-\rho} \bar{\delta}_{-1} T_{-1}^+ + e^\rho \bar{\delta}_1 T_1^+ + e^{-\rho} \bar{\beta}_{-1} T_{-1}^- + e^\rho \bar{\beta}_1 T_1^- + e^{-\rho} \bar{\eta}_{-1} W_{-1} + e^\rho \bar{\eta}_1 W_{-1} - \xi J) dx^- - \xi J dx^+ - (T_0^- + T_0^+) d\rho. \quad (3.13)$$

Note that here we have chosen the same notations to label the generators in the second copy of $sl(3|2)$. Since, A and \bar{A} are linear combinations of the tetrad (e) and the vielbein (ω) [32, 33], we can extract them from the above. The non-zero components of the tetrad turn out to be

$$\begin{aligned} e_\rho &= L_0, \\ e_+ &= \frac{1}{2}(e^{-\rho} \delta_{-1} T_{-1}^+ + e^\rho \delta_1 T_1^+ + e^{-\rho} \beta_{-1} T_{-1}^- + e^\rho \beta_1 T_1^- + e^{-\rho} \eta_{-1} W_{-1} + e^\rho \eta_1 W_1), \\ e_- &= \frac{1}{2}(e^{-\rho} \bar{\delta}_{-1} T_{-1}^+ + e^\rho \bar{\delta}_1 T_1^+ + e^{-\rho} \bar{\beta}_{-1} T_{-1}^- + e^\rho \bar{\beta}_1 T_1^- + e^{-\rho} \bar{\eta}_{-1} W_{-1} + e^\rho \bar{\eta}_1 W_{-1}). \end{aligned} \quad (3.14)$$

The metric is given by the following formula.

$$g_{\mu\nu} = \frac{1}{\epsilon_{(3|2)}} \text{str}(e_\mu e_\nu), \quad (3.15)$$

where $\epsilon_{(3|2)} = \text{str}(L_0^2) = \text{str}(T_0^+ + T_0^-)^2$. Evaluating it explicitly we obtain $\epsilon_{(3|2)} = 3/4$. By choosing this normalization we have chosen the gravitational $sl(2)$ to be the those corresponding to the generators L_\pm, L_0 . The commutation relations in (2.5) and (2.6) show that it is under these generators that all fields have well defined weights. From the super \mathcal{W}_3 conformal field theory point of view these are the modes which are part of the stress tensor of the theory. One then obtains

$$\begin{aligned} g_{\rho\rho} &= 1, \\ g_{++} &= -\frac{2}{3}(\beta_1\beta_{-1} - \frac{9}{16}\eta_1\eta_{-1} - \frac{1}{4}\delta_1\delta_{-1}), \\ g_{--} &= -\frac{2}{3}(\bar{\beta}_1\bar{\beta}_{-1} - \frac{9}{16}\bar{\eta}_1\bar{\eta}_{-1} - \frac{1}{4}\bar{\delta}_1\bar{\delta}_{-1}). \end{aligned} \quad (3.16)$$

We now demand $g_{++} = g_{--}$. This results in the following equations

$$\bar{\delta}_{\pm 1} = \zeta^{\pm 1}\delta_{\pm 1}, \quad \bar{\beta}_{\pm 1} = \zeta^{\pm 1}\beta_{\pm 1}, \quad \bar{\eta}_{\pm 1} = \zeta^{\pm 1}\eta_{\pm 1}. \quad (3.17)$$

where ζ is constant. g_{++} and g_{--} now become

$$g_{\pm\pm} = -\frac{2}{3}(\beta_1\beta_{-1} - \frac{9}{16}\eta_1\eta_{-1} - \frac{1}{4}\delta_1\delta_{-1}), \quad (3.18)$$

and g_{+-} has the form

$$g_{+-} = \frac{2}{3} \left(-\frac{1}{\zeta} (\beta_{-1}^2 - \frac{9}{16}\eta_{-1}^2 - \frac{1}{4}\delta_{-1}^2) e^{-2\rho} - \zeta (\beta_1^2 - \frac{9}{16}\eta_1^2 - \frac{1}{4}\delta_1^2) e^{2\rho} \right). \quad (3.19)$$

The metric then in terms of the (ρ, t, ϕ) coordinates is as follows.

$$\begin{aligned} ds^2 &= d\rho^2 \\ &- \frac{4}{3} \left(\zeta (\beta_1^2 - \frac{9}{16}\eta_1^2 - \frac{1}{4}\delta_1^2) e^{2\rho} + 2(\beta_1\beta_{-1} - \frac{9}{16}\eta_1\eta_{-1} - \frac{1}{4}\delta_1\delta_{-1}) \right. \\ &\quad \left. + \zeta^{-1} (\beta_{-1}^2 - \frac{9}{16}\eta_{-1}^2 - \frac{1}{4}\delta_{-1}^2) e^{-2\rho} \right) dt^2, \\ &+ \frac{4}{3} \left(\zeta (\beta_{-1}^2 - \frac{9}{16}\eta_{-1}^2 - \frac{1}{4}\delta_{-1}^2) e^{2\rho} - 2(\beta_1\beta_{-1} - \frac{9}{16}\eta_1\eta_{-1} - \frac{1}{4}\delta_1\delta_{-1}) \right. \\ &\quad \left. + \zeta^{-1} (\beta_1^2 - \frac{9}{16}\eta_1^2 - \frac{1}{4}\delta_1^2) e^{-2\rho} \right) d\phi^2. \end{aligned}$$

We now need to impose the fact that g_{tt} and $g_{\phi\phi}$ need to have a perfect square form. The results in the following equation

$$(\beta_1^2 - \frac{9}{16}\eta_1^2 - \frac{1}{2}\delta_1^2)(\beta_{-1}^2 - \frac{9}{16}\eta_{-1}^2 - \frac{1}{2}\delta_{-1}^2) = (\beta_1\beta_{-1} - \frac{9}{16}\eta_1\eta_{-1} - \frac{1}{4}\delta_1\delta_{-1})^2 \quad (3.20)$$

This imposes the conditions

$$\delta_{-1} = \alpha\delta_1, \quad \beta_{-1} = \alpha\beta_1, \quad \eta_{-1} = \alpha\eta_1. \quad (3.21)$$

Defining $\delta = \delta_1$, $\beta = \beta_1$ and $\eta = \eta_1$, the final form the metric with these conditions is

$$ds^2 = d\rho^2 - \frac{4}{3}(\beta^2 - (\frac{3}{4}\eta)^2 - (\frac{1}{2}\delta)^2) \left[\left(\sqrt{\zeta}e^\rho + \frac{\alpha}{\sqrt{\zeta}}e^{-\rho} \right)^2 dt^2 - \left(\sqrt{\zeta}e^\rho - \frac{\alpha}{\sqrt{\zeta}}e^{-\rho} \right)^2 d\phi^2 \right]. \quad (3.22)$$

By redefining ρ as $\rho \rightarrow \rho - \frac{1}{2} \log \left(\frac{\zeta}{\alpha} \right)$ we can write (3.22) as

$$ds^2 = d\rho^2 - \frac{16\alpha}{3}(\beta^2 - (\frac{3}{4}\eta)^2 - (\frac{1}{2}\delta)^2) [(\sinh^2 \rho)dt^2 - (\cosh^2 \rho)d\phi^2]. \quad (3.23)$$

From examining this metric it is easy to see that it is only for special values of the parameters $\alpha, \beta, \eta, \delta$ the metric reduces to global AdS_3 . For generic values the solution is metrically singular. The special values at which these solutions reduce to solutions studied earlier in the literature will be discussed subsequently.

Killing spinors for the higher spin conical defect

The equation for the covariantly constant spinor is given by

$$\mathcal{D}_\mu \lambda \equiv \partial_\mu \lambda + [A_\mu, \lambda] = 0, \quad (3.24)$$

where λ is given by

$$\lambda \equiv \sum_{r=-1/2}^{1/2} \epsilon^r G_r^+ + \sum_{r=-1/2}^{1/2} \tilde{\epsilon}^r G_r^- + \sum_{r=-3/2}^{3/2} \lambda^r U_r^+ + \sum_{r=-3/2}^{3/2} \tilde{\lambda}^r U_r^-. \quad (3.25)$$

From the analysis of the previous section the gauge connection for the higher conical defect is given by

$$A = (\alpha\delta e^{-\rho} T_{-1}^+ + \delta e^{\rho} T_1^+ + \alpha\beta e^{-\rho} T_1^- + \beta e^{\rho} T_{-1}^- + \alpha\eta e^{-\rho} W_{-1} + \eta e^{\rho} W_1 + \xi J) dx^+ - \xi J_0 dx^- + L_0 d\rho. \quad (3.26)$$

where $L_0 = T_0^+ + T_0^-$. We will study the supersymmetry of only one copy of the $sl(3|2)_L \times sl(3|2)_R$ Chern-Simons theory. A similar analysis can be repeated for the second copy.

Extracting out the components of the connection given in (3.26) as in (3.1) along with the exponential ρ dependence we obtain

$$\begin{aligned} j_+ &= \xi, & j_- &= -\xi \\ s_+^{-1} &= \alpha\delta e^{-\rho}, & s_+^1 &= \delta e^{\rho}, & l_\rho^0 &= 1, \\ t_+^{-1} &= \alpha\beta e^{-\rho}, & t_+^1 &= \beta e^{\rho}, \\ w_+^{-1} &= \alpha\eta e^{-\rho}, & w_+^1 &= \eta e^{\rho}, \end{aligned} \quad (3.27)$$

where, t and s are the components corresponding to the generators T^+ and T^- respectively. The equation (3.24) for the components G_r^+ and U_r^+ is given by

$$\begin{aligned}
& \partial_\mu \begin{pmatrix} \epsilon^{-1/2} \\ \epsilon^{1/2} \\ \lambda^{-3/2} \\ \lambda^{-1/2} \\ \lambda^{1/2} \\ \lambda^{3/2} \end{pmatrix} \\
& + \frac{1}{4} \begin{pmatrix} 4j_0+2l_0 & \frac{16}{3}s_{-1}+\frac{4}{3}t_{-1} & 4i(s_{-1}-t_{-1})+3w_1 & 0 & \frac{4i}{3}(s_{-1}-t_{-1})+3w_{-1} & 0 \\ \frac{16}{3}s_{-1}-\frac{4}{3}t_{-1} & 4j_0-2l_0 & 0 & \frac{4i}{3}(s_{-1}-t_{-1})-3w_1 & 0 & 4i(s_{-1}-t_{-1})-3w_1 \\ \frac{8i}{3}(s_{-1}-t_{-1})+2w_{-1} & 0 & 4j_0+6l_0 & -\frac{8}{3}s_{-1}-\frac{4}{3}t_{-1}+2iw_{-1} & 0 & 0 \\ 0 & \frac{8i}{3}(s_{-1}-t_{-1})-6w_{-1} & 8s_1+4t_1-6iw_1 & 4j_0+2l_0 & -\frac{16}{3}s_{-1}-\frac{8}{3}t_{-1} & 0 \\ \frac{8i}{3}(s_{-1}-t_{-1})+6w_1 & 0 & 0 & \frac{16}{3}s_1+\frac{8}{3}t_1 & 4j_0-2l_0 & -8s_{-1}-4t_{-1}-6iw_{-1} \\ 0 & \frac{8i}{3}(s_{-1}-t_{-1})-2w_1 & 0 & 0 & \frac{8}{3}s_1+\frac{4}{3}t_1+2iw_1 & 4j_0-6l_0 \end{pmatrix}_\mu \\
& \times \begin{pmatrix} \epsilon^{-1/2} \\ \epsilon^{1/2} \\ \lambda^{-3/2} \\ \lambda^{-1/2} \\ \lambda^{1/2} \\ \lambda^{3/2} \end{pmatrix} = 0. \tag{3.28}
\end{aligned}$$

The x^+ dependence of the column spinor above is determined by the eigenvalues of the $\mathcal{R}^{-1}(\rho)\mathbb{M}_+\mathcal{R}(\rho)$ matrix. The solutions are of the form

$$\begin{aligned}
& \begin{pmatrix} \epsilon^{-1/2} \\ \epsilon^{1/2} \\ \lambda^{-3/2} \\ \lambda^{-1/2} \\ \lambda^{1/2} \\ \lambda^{3/2} \end{pmatrix} \\
& = \mathcal{R}(\rho) e^{\xi(x_- - x_+)} (d_1 e^{i\sqrt{\alpha}\delta x_+} \mathbf{z}_1 + d_2 e^{-i\sqrt{\alpha}\delta x_+} \mathbf{z}_2 \\
& \quad + d_3 e^{i\sqrt{\alpha}\left(\delta+2\left(\beta^2-\left(\frac{3}{4}\eta\right)^2\right)^{1/2}\right)x_+} \mathbf{z}_3 + d_4 e^{-i\sqrt{\alpha}\left(\delta+2\left(\beta^2-\left(\frac{3}{4}\eta\right)^2\right)^{1/2}\right)x_+} \mathbf{z}_4 \\
& \quad + d_5 e^{i\sqrt{\alpha}\left(\delta-2\left(\beta^2-\left(\frac{3}{4}\eta\right)^2\right)^{1/2}\right)x_+} \mathbf{z}_5 + d_6 e^{-i\sqrt{\alpha}\left(\delta-2\left(\beta^2-\left(\frac{3}{4}\eta\right)^2\right)^{1/2}\right)x_+} \mathbf{z}_6). \tag{3.29}
\end{aligned}$$

The matrix \mathcal{R} has the ρ dependence as in (3.6).

Now re-expressing x_+ and x_- in terms of the co-ordinates t and ϕ allows us to obtain the condition under which any of the above Killing spinor is periodic. The possibilities are the following:

$$2\xi = \pm i\sqrt{\alpha}\delta + in, \tag{3.30}$$

$$2\xi = \pm i\sqrt{\alpha}\left(\delta \pm 2\left(\beta^2 - \left(\frac{3}{4}\eta\right)^2\right)^{1/2}\right) + in. \tag{3.31}$$

where n is any integer.

We have also examined the Killing spinor equation for the G^- and U^- components. On repeating the same analysis we have seen that the component $u(1)$ gauge

field, ξ has to be complex in order to impose proper periodicity requirements. Since this is not allowed we conclude that there are no Killing spinors corresponding to conjugates of the G^- , U^- charges.

To relate to known solutions, we will obtain the special values of the parameter space at which the higher spin conical defect reduces to solutions studied earlier in the literature.

Supersymmetry of conical defects in $sl(2)$

Embedding the conical defect solution only in the $sl(2) \oplus u(1)$ sub-algebra we have the following gauge connections

$$\begin{aligned} A &= \left(e^\rho T_1^+ + \frac{\gamma}{4} e^{-\rho} T_{-1}^+ \right) dx^+ + T_0^+ d\rho + 2\xi J d\phi, \\ \bar{A} &= - \left(e^\rho T_{-1}^+ + \frac{\gamma}{4} e^{-\rho} T_1^+ \right) dx^+ - T_0^+ d\rho + 2\xi J d\phi. \end{aligned} \quad (3.32)$$

Note that this gauge connection is a special case of the higher spin conical defect with $\alpha = \gamma/4$, $\delta = 1$ and $\beta = \eta = 0$.

One can perform a gauge transformation $A \rightarrow U^-(A + d)U$ with $U = e^{\rho T_0^-}$ on the connection (3.32). The new connections are then of the form

$$\begin{aligned} A &= \left(e^\rho T_1^+ + \frac{\gamma}{4} e^{-\rho} T_{-1}^+ \right) dx^+ + (T_0^+ + T_0^-) d\rho + 2\xi J d\phi, \\ \bar{A} &= - \left(e^\rho T_{-1}^+ + \frac{\gamma}{4} e^{-\rho} T_1^+ \right) dx^+ - (T_0^+ + T_0^-) d\rho + 2\xi J d\phi. \end{aligned} \quad (3.33)$$

where, for \bar{A} we have used the transformation by $U = e^{-\rho T_0^-}$. Now the gauge connections are of the general form given in (3.1).

The equation for the covariantly constant spinor is

$$\mathcal{D}_\mu \lambda \equiv \partial_\mu \lambda + [A_\mu, \lambda] = 0, \quad (3.34)$$

where λ is given by

$$\lambda \equiv \sum_{r=-1/2}^{1/2} \epsilon^r G_r^+ + \sum_{r=-1/2}^{1/2} \tilde{\epsilon}^r G_r^- + \sum_{r=-3/2}^{3/2} \lambda^r U_r^+ + \sum_{r=-3/2}^{3/2} \tilde{\lambda}^r U_r^-. \quad (3.35)$$

The analysis for the Killing spinor performed for the case of the higher spin conical defect can be repeated. The solutions of the components of the generators G^\pm, U^\pm

are of the form

$$\begin{aligned} & \begin{pmatrix} \epsilon^{-1/2} \\ \epsilon^{1/2} \\ \lambda^{-3/2} \\ \lambda^{-1/2} \\ \lambda^{1/2} \\ \lambda^{3/2} \end{pmatrix}_{\pm} \\ &= \mathcal{R}(\rho) e^{\xi(x_- - x_+)} \left(e^{i\frac{\sqrt{\gamma}}{2}x_+} (d_1\mathbf{z}_1 + d_2\mathbf{z}_2 + d_3\mathbf{z}_3) + e^{-i\frac{\sqrt{\gamma}}{2}x_+} (d_4\mathbf{z}_4 + d_5\mathbf{z}_5 + d_6\mathbf{z}_6) \right)_{\pm} \end{aligned} \quad (3.36)$$

\mathbf{z}_i are the eigenvectors of the 6×6 matrices which appear in the Killing spinor equation. The ρ dependence is contained in the matrix $\mathcal{R}(\rho)$ given in (3.6).

Imposing periodicity on the Killing spinor, we obtain

$$\xi = \pm i \frac{\sqrt{\gamma}}{4} + in. \quad (3.37)$$

Note that this condition coincides with the condition found for Killing spinors in [24]. Since there is a pair of eigenvalues with degeneracy 3, we will in general have 3 Killing spinors which will satisfy the periodicity condition.

Supersymmetry of Anti-deSitter space in $sl(2)$

For the case of AdS_3 one can perform the same analysis with $\gamma = 1$. As expected, it can be seen that one does not require the $u(1)$ gauge field and one obtains anti-periodic Killing spinors. The solution for the Killing spinors for this case is

$$\begin{aligned} & \begin{pmatrix} \epsilon^{-1/2} \\ \epsilon^{1/2} \\ \lambda^{-3/2} \\ \lambda^{-1/2} \\ \lambda^{1/2} \\ \lambda^{3/2} \end{pmatrix}_{\pm} \\ &= \mathcal{R}(\rho) \left(e^{i\frac{x_{\pm}}{2}} (d_1\mathbf{z}_1 + d_2\mathbf{z}_2 + d_3\mathbf{z}_3) + e^{-i\frac{x_{\pm}}{2}} (d_4\mathbf{z}_4 + d_5\mathbf{z}_5 + d_6\mathbf{z}_6) \right)_{\pm}. \end{aligned} \quad (3.38)$$

The ρ dependence of the Killing spinor remains the same as the one for the conical defect. This AdS_3 in $sl(2)$ admits 6 anti-periodic Killing spinors.

Supersymmetry of conical defects in the gravitational $sl(2)$

We now write down the metric for the conical defect embedded in the gravitational $sl(2)$ generated by the L_m generators. In Fefferman-Graham coordinates this metric is given by

$$ds^2 = d\rho^2 - \left(e^{\rho} + \frac{\gamma}{4} e^{-\rho} \right)^2 dt^2 + \left(e^{\rho} - \frac{\gamma}{4} e^{-\rho} \right)^2 d\phi^2. \quad (3.39)$$

This can be equivalently written in terms of the gauge connections

$$A = \left(e^\rho L_1 + \frac{\gamma}{4} e^{-\rho} L_{-1} \right) dx^+ + L_0 d\rho + 2\xi J_0 d\phi, \quad (3.40)$$

$$\bar{A} = - \left(e^\rho L_{-1} + \frac{\gamma}{4} e^{-\rho} L_1 \right) dx^+ - L_0 d\rho + 2\xi J_0 d\phi. \quad (3.41)$$

Note that this connection is a special case of (3.26) with $\beta = \delta, \eta = 0, \zeta = 1$ and $\alpha = \frac{\gamma}{4}$. These connections and the metric reduce to that of global AdS by setting $\gamma = 1$ and $\xi = 0$. The non-zero components of the connection are

$$l_+^1 = e^\rho, \quad l_+^{-1} = \frac{\gamma}{4} e^{-\rho}, \quad l_\rho^0 = 1, \quad j_+^0 = \xi, \quad j_-^0 = -\xi. \quad (3.42)$$

The equation for the covariantly conserved spinor is given by

$$\mathcal{D}_\mu \lambda \equiv \partial_\mu \lambda + [A_\mu, \lambda] = 0, \quad (3.43)$$

where λ is given by

$$\lambda \equiv \sum_{r=-1/2}^{1/2} \epsilon^r G_r^+ + \sum_{r=-1/2}^{1/2} \tilde{\epsilon}^r G_r^- + \sum_{r=-3/2}^{3/2} \lambda^r U_r^+ + \sum_{r=-3/2}^{3/2} \tilde{\lambda}^r U_r^-. \quad (3.44)$$

For the connection given in (3.40) the Killing spinor equations for the G_r^\pm and U_r^\pm decouple. The equations for the $G_{\pm 1/2}^+$ components in matrix form is given by

$$\partial_\mu \begin{pmatrix} \epsilon^{-1/2} \\ \epsilon^{1/2} \end{pmatrix} + \begin{pmatrix} \frac{1}{2}(2j^0 + l^0)_\mu & -l_\mu^{-1} \\ l_\mu^1 & \frac{1}{2}(2j^0 - l^0)_\mu \end{pmatrix} \begin{pmatrix} \epsilon^{-1/2} \\ \epsilon^{1/2} \end{pmatrix} = 0. \quad (3.45)$$

The solutions are given by

$$\begin{aligned} \begin{pmatrix} \epsilon^{-1/2} \\ \epsilon^{1/2} \end{pmatrix} &= \mathcal{R}_1(\rho) e^{\xi(x_- - x_+)} (c_1 e^{i\frac{\sqrt{\gamma}}{2}x_+} \mathbf{y}_1 + c_2 e^{-i\frac{\sqrt{\gamma}}{2}x_+} \mathbf{y}_2) \\ &= \mathcal{R}_1(\rho) e^{-2\xi\phi} (c_1 e^{i\frac{\sqrt{\gamma}}{2}(t+\phi)} \mathbf{y}_1 + c_2 e^{-i\frac{\sqrt{\gamma}}{2}(t+\phi)} \mathbf{y}_2), \end{aligned} \quad (3.46)$$

where, $\mathbf{y}_{1,2}$ are the eigenvectors of the matrix $\mathcal{R}_1^{-1} \mathbb{M}_+ \mathcal{R}_1$. Here \mathbb{M}_μ is the matrix which appears in the equation (3.45) and $\mathcal{R}_1(\rho)$ is a diagonal matrix with the following ρ dependence

$$\mathcal{R}_1(\rho) = \begin{pmatrix} e^{-\rho/2} & 0 \\ 0 & e^{\rho/2} \end{pmatrix}. \quad (3.47)$$

The equations for the U_r^+ generators are

$$\partial_\mu \begin{pmatrix} \lambda^{-3/2} \\ \lambda^{-1/2} \\ \lambda^{1/2} \\ \lambda^{3/2} \end{pmatrix} + \begin{pmatrix} \frac{1}{2}(2j^0 + 3l^0)_\mu & -l_\mu^{-1} & 0 & 0 \\ 3l_\mu^1 & \frac{1}{2}(2j^0 + l^0)_\mu & -2l_\mu^{-1} & 0 \\ 0 & 2l_\mu^1 & \frac{1}{2}(2j^0 - l^0)_\mu & -3l_\mu^{-1} \\ 0 & 0 & l_\mu^1 & \frac{1}{2}(2j^0 - 3l^0)_\mu \end{pmatrix} \begin{pmatrix} \lambda^{-3/2} \\ \lambda^{-1/2} \\ \lambda^{1/2} \\ \lambda^{3/2} \end{pmatrix} = 0. \quad (3.48)$$

The solutions are given by

$$\begin{aligned}
& \begin{pmatrix} \lambda^{-3/2} \\ \lambda^{-1/2} \\ \lambda^{1/2} \\ \lambda^{3/2} \end{pmatrix} \\
&= \mathcal{R}_2(\rho) e^{\xi(x_- - x_+)} (d_1 e^{i\frac{\sqrt{\gamma}}{2}x_+} \mathbf{z}_1 + d_2 e^{-i\frac{\sqrt{\gamma}}{2}x_+} \mathbf{z}_2 \\
&\quad + d_3 e^{i\frac{3\sqrt{\gamma}}{2}x_+} \mathbf{z}_3 + d_4 e^{-i\frac{3\sqrt{\gamma}}{2}x_+} \mathbf{z}_4), \\
&= \mathcal{R}_2(\rho) e^{-2\xi\phi} (d_1 e^{i\frac{\sqrt{\gamma}}{2}(t+\phi)} \mathbf{z}_1 + d_2 e^{-i\frac{\sqrt{\gamma}}{2}(t+\phi)} \mathbf{z}_2 \\
&\quad + d_3 e^{i\frac{3\sqrt{\gamma}}{2}(t+\phi)} \mathbf{z}_3 + d_4 e^{-i\frac{3\sqrt{\gamma}}{2}(t+\phi)} \mathbf{z}_4). \tag{3.49}
\end{aligned}$$

The matrix \mathcal{R}_2 has the ρ dependence

$$\mathcal{R}_2(\rho) = \begin{pmatrix} e^{-3\rho/2} & 0 & 0 & 0 \\ 0 & e^{-\rho/2} & 0 & 0 \\ 0 & 0 & e^{\rho/2} & 0 \\ 0 & 0 & 0 & e^{3\rho/2} \end{pmatrix}. \tag{3.50}$$

We thus get 6 independent Killing spinors. The conditions which we obtain on demanding periodicity of the spinor is

$$\xi = \pm i\frac{\gamma}{4} + in, \quad \text{or} \quad \xi = \pm 3i\frac{\gamma}{4} + in. \tag{3.51}$$

Thus, on embedding this conical defect in the $sl(2)$ corresponding to L_0, L_{\pm} we see that there are 4 eigenvalues out of which there are 2 doubly degenerate ones. The doubly degenerate ones obey the condition $\xi = \pm i\frac{\gamma}{4} + in$. These match with that given in (3.37) which also agrees with [24].

The Killing spinor equations for the G_r^- and U_r^- components of also form a set of 6 coupled equations. These equations are the same as the above with the replacement $j_0 \rightarrow -j_0$ or $\xi \rightarrow -(-\xi)$. Thus, they admit same solutions as given in (3.46) and (3.49) with different arbitrary constants

Supersymmetry of anti-de Sitter space in the gravitational $sl(2)$

Let us now turn to the case of global AdS_3 embedded in the gravitational $sl(2)$. This is a special case of the conical spaces embedded in the gravitational $sl(2)$ with $\gamma = 1, \xi = 0$. The metric in terms of the Fefferman-Graham coordinates is

$$ds^2 = d\rho^2 - \left(e^{\rho} + \frac{1}{4}e^{-\rho} \right)^2 dt^2 + \left(e^{\rho} - \frac{1}{4}e^{-\rho} \right)^2 d\phi^2. \tag{3.52}$$

This can be equivalently written in terms of the gauge connections

$$\begin{aligned} A &= \left(e^\rho L_1 + \frac{1}{4} e^{-\rho} L_{-1} \right) dx^+ + L_0 d\rho, \\ \bar{A} &= - \left(e^\rho L_{-1} + \frac{1}{4} e^{-\rho} L_1 \right) dx^+ - L_0 d\rho. \end{aligned} \quad (3.53)$$

The solutions for the Killing spinors are given by

$$\begin{aligned} \begin{pmatrix} \epsilon^{-1/2} \\ \epsilon^{1/2} \end{pmatrix} &= \mathcal{R}_1(\rho) (c_1 e^{\frac{i}{2}x^+} \mathbf{y}_1 + c_2 e^{-\frac{i}{2}x^+} \mathbf{y}_2), \\ &= \mathcal{R}_1(\rho) (c_1 e^{\frac{i}{2}(t+\phi)} \mathbf{y}_1 + c_2 e^{-\frac{i}{2}(t+\phi)} \mathbf{y}_2), \end{aligned} \quad (3.54)$$

and

$$\begin{aligned} \begin{pmatrix} \lambda^{-3/2} \\ \lambda^{-1/2} \\ \lambda^{1/2} \\ \lambda^{3/2} \end{pmatrix} &= \mathcal{R}_2(\rho) (d_1 e^{\frac{i}{2}x^+} \mathbf{z}_1 + d_2 e^{-\frac{i}{2}x^+} \mathbf{z}_2 + d_3 e^{\frac{3i}{2}x^+} \mathbf{z}_3 + d_4 e^{-\frac{3i}{2}x^+} \mathbf{z}_4), \\ &= \mathcal{R}_2(\rho) (d_1 e^{\frac{i}{2}(t+\phi)} \mathbf{z}_1 + d_2 e^{-\frac{i}{2}(t+\phi)} \mathbf{z}_2 + d_3 e^{\frac{3i}{2}(t+\phi)} \mathbf{z}_3 + d_4 e^{-\frac{3i}{2}(t+\phi)} \mathbf{z}_4). \end{aligned} \quad (3.55)$$

We obtain 6 independent Killing spinors which are anti-periodic corresponding to the G_r^+ and U_r^+ generators. Similarly performing the same analysis it is easy to see that one obtains 6 independent anti-periodic Killing spinors corresponding to the G_r^- and U_r^- generators.

The holonomy of global AdS_3 around the angular direction ϕ embedded in the gravitational $sl(2)$ can be shown to be trivial and therefore the solution is smooth. Thus this background corresponds to the supersymmetric vacuum in the Neveu-Schwarz sector of the dual CFT.

3.3 The BTZ black hole

The BTZ black hole in $sl(2)$

We now examine the supersymmetry of the connection corresponding to that of the BTZ black hole embedded in the $sl(2)$ part of bosonic algebra $sl(3) \oplus sl(2) \oplus u(1)$. The connections are given by

$$\begin{aligned} A &= \left(e^\rho T_1^+ - \frac{2\pi}{k} \mathcal{L} e^{-\rho} T_{-1}^+ \right) dx^+ + T_0^+ d\rho, \\ \bar{A} &= - \left(e^\rho T_{-1}^+ - \frac{2\pi}{k} \bar{\mathcal{L}} e^{-\rho} T_1^+ \right) dx^- - T_0^+ d\rho, \end{aligned} \quad (3.56)$$

where

$$\mathcal{L} = \frac{M - \hat{J}}{4\pi} \quad , \quad \bar{\mathcal{L}} = \frac{M + \hat{J}}{4\pi} \quad (3.57)$$

We shall make a gauge transformation $A \rightarrow U^-(A + d)U$ to the above connections with $U = e^{\rho T_0^-}$ for A and $U = e^{-\rho T_0^-}$ for \bar{A} . This gives

$$\begin{aligned} A &= \left(e^{\rho T_1^+} - \frac{2\pi}{k} \mathcal{L} e^{-\rho T_{-1}^+} \right) dx^+ + (T_0^+ + T_0^-) d\rho, \\ \bar{A} &= - \left(e^{\rho T_{-1}^+} - \frac{2\pi}{k} \bar{\mathcal{L}} e^{-\rho T_1^+} \right) dx^- - (T_0^+ + T_0^-) d\rho. \end{aligned} \quad (3.58)$$

Now the connection is of the general form given by (3.1).

For the extremal case ($M = \hat{J}$) we have, $\mathcal{L} = 0$ and therefore the connection A reduces to

$$A = e^{\rho T_1^+} dx^+ + (T_0^+ + T_0^-) d\rho. \quad (3.59)$$

The equation for the Killing spinor is given by

$$\mathcal{D}_\mu \lambda \equiv \partial_\mu \lambda + [A_\mu, \lambda] = 0, \quad (3.60)$$

where λ is expanded as

$$\lambda \equiv \sum_{r=-1/2}^{1/2} \epsilon^r G_r^+ + \sum_{r=-1/2}^{1/2} \tilde{\epsilon}^r G_r^- + \sum_{r=-3/2}^{3/2} \lambda^r U_r^+ + \sum_{r=-3/2}^{3/2} \tilde{\lambda}^r U_r^-. \quad (3.61)$$

Writing the equation given in (3.60) explicitly we obtain the following equation for the ρ direction

$$\partial_\rho \begin{pmatrix} \epsilon^{-1/2} \\ \epsilon^{1/2} \\ \lambda^{-3/2} \\ \lambda^{-1/2} \\ \lambda^{1/2} \\ \lambda^{3/2} \end{pmatrix} + \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{3}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{3}{2} \end{pmatrix} \begin{pmatrix} \epsilon^{-1/2} \\ \epsilon^{1/2} \\ \lambda^{-3/2} \\ \lambda^{-1/2} \\ \lambda^{1/2} \\ \lambda^{3/2} \end{pmatrix} = 0. \quad (3.62)$$

Similarly the equation for the $+$ direction is given by

$$\partial_+ \begin{pmatrix} \epsilon^{-1/2} \\ \epsilon^{1/2} \\ \lambda^{-3/2} \\ \lambda^{-1/2} \\ \lambda^{1/2} \\ \lambda^{3/2} \end{pmatrix} + \begin{pmatrix} 0 & 0 & -i & 0 & 0 & 0 \\ -\frac{1}{3} & 0 & 0 & -\frac{i}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ -\frac{2i}{3} & 0 & 0 & \frac{2}{3} & 0 & 0 \\ 0 & -\frac{2i}{3} & 0 & 0 & \frac{1}{3} & 0 \end{pmatrix} \begin{pmatrix} \epsilon^{-1/2} \\ \epsilon^{1/2} \\ \lambda^{-3/2} \\ \lambda^{-1/2} \\ \lambda^{1/2} \\ \lambda^{3/2} \end{pmatrix} = 0. \quad (3.63)$$

The solutions of these equations are of the form

$$\begin{pmatrix} \epsilon^{-1/2} \\ \epsilon^{1/2} \\ \lambda^{-3/2} \\ \lambda^{-1/2} \\ \lambda^{1/2} \\ \lambda^{3/2} \end{pmatrix} = \begin{pmatrix} c_3 e^{-\rho/2 - i\pi/2} \\ \frac{c_2}{2} e^{\rho/2 - i\pi/2} \\ 0 \\ c_3 e^{-\rho/2} \\ c_2 e^{\rho/2} \\ c_1 e^{3\rho/2} \end{pmatrix}. \quad (3.64)$$

Thus there are 3 linearly independent Killing spinors corresponding to the supercharges with positive \hat{J} charge for the extremal BTZ embedded in the $sl(3|2)$ theory.

The BTZ black hole in gravitational $sl(2)$

The connection of the BTZ black hole embedded in the gravitational $sl(2)$ is given by

$$\begin{aligned} A &= \left(e^\rho L_1 - \frac{2\pi}{k} \mathcal{L} e^{-\rho} L_{-1} \right) dx^+ + L_0 d\rho, \\ \bar{A} &= - \left(e^\rho L_{-1} - \frac{2\pi}{k} \bar{\mathcal{L}} e^{-\rho} L_1 \right) dx^- - L_0 d\rho, \end{aligned} \quad (3.65)$$

where

$$\mathcal{L} = \frac{M - \hat{J}}{4\pi}, \quad \bar{\mathcal{L}} = \frac{M + \hat{J}}{4\pi}. \quad (3.66)$$

Substituting $M = \hat{J}$ for the extremal BTZ the connection reduces to

$$A = e^\rho L_1 dx^+ + L_0 d\rho. \quad (3.67)$$

The equation for the covariantly constant spinor is given by

$$\mathcal{D}_\mu \lambda \equiv \partial_\mu \lambda + [A_\mu, \lambda] = 0, \quad (3.68)$$

where λ is expanded as

$$\lambda \equiv \sum_{r=-1/2}^{1/2} \epsilon^r G_r^+ + \sum_{r=-1/2}^{1/2} \bar{\epsilon}^r G_r^- + \sum_{r=-3/2}^{3/2} \lambda^r U_r^+ + \sum_{r=-3/2}^{3/2} \tilde{\lambda}^r U_r^-. \quad (3.69)$$

Writing out the Killing spinor equations for the G_r^+ generators we obtain

$$\partial_\mu \begin{pmatrix} \epsilon^{-1/2} \\ \epsilon^{1/2} \end{pmatrix} + \begin{pmatrix} \frac{1}{2}(2j^0 + l^0)_\mu & -l_\mu^{-1} \\ -l_\mu^1 & \frac{1}{2}(2j^0 - l^0)_\mu \end{pmatrix} \begin{pmatrix} \epsilon^{-1/2} \\ \epsilon^{1/2} \end{pmatrix} = 0. \quad (3.70)$$

Similarly the equations for U^+ generators are given by

$$\partial_\mu \begin{pmatrix} \lambda^{-3/2} \\ \lambda^{-1/2} \\ \lambda^{1/2} \\ \lambda^{3/2} \end{pmatrix} + \begin{pmatrix} \frac{3}{2}l_\mu^0 & 0 & 0 & 0 \\ 3l_\mu^1 & \frac{3}{2}l_\mu^0 & 0 & 0 \\ 0 & 2l_\mu^1 & -\frac{3}{2}l_\mu^0 & 0 \\ 0 & 0 & l_\mu^1 & -\frac{3}{2}l_\mu^0 \end{pmatrix} \begin{pmatrix} \lambda^{-3/2} \\ \lambda^{-1/2} \\ \lambda^{1/2} \\ \lambda^{3/2} \end{pmatrix} = 0. \quad (3.71)$$

The equations for $\tilde{\epsilon}$ and $\tilde{\lambda}$ are identical to these but with the replacements $\epsilon \rightarrow \tilde{\epsilon}$ and $\lambda \rightarrow \tilde{\lambda}$. The Killing spinor which is periodic in the angular direction is given by

$$\begin{pmatrix} \epsilon^{-1/2} \\ \epsilon^{1/2} \end{pmatrix} = \begin{pmatrix} 0 \\ C e^{\rho/2} \end{pmatrix}, \quad \begin{pmatrix} \tilde{\epsilon}^{-1/2} \\ \tilde{\epsilon}^{1/2} \end{pmatrix} = \begin{pmatrix} 0 \\ \tilde{C} e^{\rho/2} \end{pmatrix} \quad (3.72)$$

$$\begin{pmatrix} \lambda^{-3/2} \\ \lambda^{-1/2} \\ \lambda^{1/2} \\ \lambda^{3/2} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ D e^{3\rho/2} \end{pmatrix}, \quad \begin{pmatrix} \tilde{\lambda}^{-3/2} \\ \tilde{\lambda}^{-1/2} \\ \tilde{\lambda}^{1/2} \\ \tilde{\lambda}^{3/2} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \tilde{D} e^{3\rho/2} \end{pmatrix} \quad (3.73)$$

The solution given in (3.72) matches with that obtained by [29]. Thus the extremal BTZ embedded in the gravitational $sl(2)$ admits 2 independent Killing spinors corresponding to the G^+, U^+ generators.

3.4 Higher spin black holes

Now we will study the supersymmetry of black holes with spin-3 charge recently constructed in [18]. The connections are given by

$$\begin{aligned} A = & \left(e^\rho T_1^- - \frac{2\pi}{k} \mathcal{L} e^{-\rho} T_{-1}^- + \frac{\pi}{2k\sigma} \mathcal{W} e^{-2\rho} W_{-2} \right) dx^+ \\ & + \mu \left(e^{2\rho} W_{-2} - \frac{4\pi \mathcal{L}}{k} W_0 + \frac{4\pi^2 \mathcal{L}^2}{k^2} e^{-2\rho} W_2 + \frac{4\pi \mathcal{W}}{k} e^{-\rho} T_{-1}^- \right) dx^- + 2\xi J d\phi + L_0 d\rho, \end{aligned} \quad (3.74)$$

$$\begin{aligned} \bar{A} = & - \left(e^\rho T_1^- - \frac{2\pi}{k} \bar{\mathcal{L}} e^{-\rho} T_{-1}^- + \frac{\pi}{2k\sigma} \bar{\mathcal{W}} e^{-2\rho} W_{-2} \right) dx^+ \\ & - \bar{\mu} \left(e^{2\rho} W_{-2} - \frac{4\pi \bar{\mathcal{L}}}{k} W_0 + \frac{4\pi^2 \bar{\mathcal{L}}^2}{k^2} e^{-2\rho} W_2 + \frac{4\pi \bar{\mathcal{W}}}{k} e^{-\rho} T_{-1}^- \right) dx^- + 2\xi J d\phi - L_0 d\rho, \end{aligned} \quad (3.75)$$

where $L_0 = T_0^+ + T_0^-$ and $\sigma = (3/4)^2$. These differ from the connection of [18] by a gauge transformation $U = e^{\rho T_0^+}$ and also contains a gauge field in the $u(1)$.

We shall consider the supersymmetry of the black hole with $\mathcal{W} = 0$ and $\mu = 0$ but $\bar{\mathcal{W}} \neq 0$ and $\bar{\mu} \neq 0$. Imposing this condition is analogous to imposing the extremality condition for the case of the BTZ black hole. The equation for the Killing spinor is given by

$$\mathcal{D}_\mu \lambda \equiv \partial_\mu \lambda + [A_\mu, \lambda] = 0, \quad (3.76)$$

and λ is expanded as

$$\lambda \equiv \sum_{r=-1/2}^{1/2} \epsilon^r G_r^+ + \sum_{r=-1/2}^{1/2} \tilde{\epsilon}^r G_r^- + \sum_{r=-3/2}^{3/2} \lambda^r U_r^+ + \sum_{r=-3/2}^{3/2} \tilde{\lambda}^r U_r^-. \quad (3.77)$$

Written in matrix form the equation given in (3.76) reads

$$\partial_\rho \begin{pmatrix} \epsilon^{-1/2} \\ \epsilon^{1/2} \\ \lambda^{-3/2} \\ \lambda^{-1/2} \\ \lambda^{1/2} \\ \lambda^{3/2} \end{pmatrix} + \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{3}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{3}{2} \end{pmatrix} \begin{pmatrix} \epsilon^{-1/2} \\ \epsilon^{1/2} \\ \lambda^{-3/2} \\ \lambda^{-1/2} \\ \lambda^{1/2} \\ \lambda^{3/2} \end{pmatrix} = 0, \quad (3.78)$$

$$\partial_+ \begin{pmatrix} \epsilon^{-1/2} \\ \epsilon^{1/2} \\ \lambda^{-3/2} \\ \lambda^{-1/2} \\ \lambda^{1/2} \\ \lambda^{3/2} \end{pmatrix} + \begin{pmatrix} \xi & \frac{8\mathcal{L}\pi}{3k} & i & 0 & -\frac{2i\mathcal{L}\pi}{3k} & 0 \\ \frac{4}{3} & \xi & 0 & \frac{i}{3} & 0 & -\frac{2i\mathcal{L}\pi}{k} \\ -\frac{4i\mathcal{L}\pi}{3k} & 0 & \xi & \frac{4\mathcal{L}\pi}{3k} & 0 & 0 \\ 0 & -\frac{4i\mathcal{L}\pi}{3k} & 2 & \xi & \frac{8\mathcal{L}\pi}{3k} & 0 \\ \frac{2i}{3} & 0 & 0 & \frac{4}{3} & \xi & \frac{4\mathcal{L}\pi}{k} \\ 0 & \frac{2i}{3} & 0 & 0 & \frac{2}{3} & \xi \end{pmatrix} \begin{pmatrix} \epsilon^{-1/2} \\ \epsilon^{1/2} \\ \lambda^{-3/2} \\ \lambda^{-1/2} \\ \lambda^{1/2} \\ \lambda^{3/2} \end{pmatrix} = 0. \quad (3.79)$$

$$\partial_- \begin{pmatrix} \epsilon^{-1/2} \\ \epsilon^{1/2} \\ \lambda^{-3/2} \\ \lambda^{-1/2} \\ \lambda^{1/2} \\ \lambda^{3/2} \end{pmatrix} + \begin{pmatrix} -\xi & 0 & 0 & 0 & 0 & 0 \\ 0 & -\xi & 0 & 0 & 0 & 0 \\ 0 & 0 & -\xi & 0 & 0 & 0 \\ 0 & 0 & 0 & -\xi & 0 & 0 \\ 0 & 0 & 0 & 0 & -\xi & 0 \\ 0 & 0 & 0 & 0 & 0 & -\xi \end{pmatrix} \begin{pmatrix} \epsilon^{-1/2} \\ \epsilon^{1/2} \\ \lambda^{-3/2} \\ \lambda^{-1/2} \\ \lambda^{1/2} \\ \lambda^{3/2} \end{pmatrix} = 0. \quad (3.80)$$

The solutions to these equations are given by

$$\begin{pmatrix} \epsilon^{-1/2} \\ \epsilon^{1/2} \\ \lambda^{-3/2} \\ \lambda^{-1/2} \\ \lambda^{1/2} \\ \lambda^{3/2} \end{pmatrix} = \mathcal{R}(\rho) f_+(x_+) f_-(x_-), \quad (3.81)$$

where, $\mathcal{R}(\rho)$ is defined in (3.6) and

$$f_+(x_+) = e^{-(2\sqrt{\frac{2\pi\mathcal{L}}{k}} + \xi)x_+} (c_1 \mathbf{y}_1 + c_2 \mathbf{y}_2) + e^{-(2\sqrt{\frac{2\pi\mathcal{L}}{k}} + \xi)x_+} (c_3 \mathbf{y}_3 + c_4 \mathbf{y}_4) + e^{-\xi x_+} (c_4 \mathbf{y}_4 + c_5 \mathbf{y}_5). \quad (3.82)$$

\mathbf{y}_i are the eigenvectors of the matrix that appears in the + component of the Killing spinor equation. As usual the x_- dependence is given by

$$f_-(x_-) = e^{\xi x_-}. \quad (3.83)$$

The value of the $u(1)$ field for which we get the proper periodicity of the spinor is

$$\xi = \pm \sqrt{\frac{2\pi\mathcal{L}}{k}}, \quad \text{or} \quad \xi = i\frac{n}{2}. \quad (3.84)$$

From degeneracy of the eigenvalues in (3.82) we see that in general we can have two Killing spinors for a given ξ satisfying any one of the conditions in (3.84).

A new higher spin black hole

We shall now try to generalize the gauge connection (3.74) by including terms which involve the $sl(2)$ corresponding to the T_{\pm}^-, T_0^- generators. This solution is same as the one given in (3.74) but with the $sl(2)$ connections of BTZ the black hole added to it. It may thus admit a notion of the horizon. The connection is given as follows and we have verified that it obeys the flatness conditions.

$$\begin{aligned} A = & \left(e^{\rho}T_1^- - \frac{2\pi}{k}\mathcal{L}_1e^{-\rho}T_{-1}^- + \frac{\pi}{2k\sigma}\mathcal{W}e^{-2\rho}W_{-2} + e^{\rho}T_1^+ - \frac{2\pi}{k}\mathcal{L}_2e^{-\rho}T_{-1}^+ \right) dx^+ \\ & + \mu \left(e^{2\rho}W_2 - \frac{4\pi\mathcal{L}_1}{k}W_0 + \frac{4\pi^2\mathcal{L}_1^2}{k^2}e^{-2\rho}W_{-2} + \frac{4\pi\mathcal{W}}{k}e^{-\rho}T_{-1}^- \right) dx^- + 2\xi Jd\phi, \\ & + (T_0^- + T_0^+)d\rho \end{aligned} \quad (3.85)$$

$$\begin{aligned} \bar{A} = & - \left(e^{\rho}T_1^- - \frac{2\pi}{k}\bar{\mathcal{L}}_1e^{-\rho}T_{-1}^- + \frac{\pi}{2k\sigma}\bar{\mathcal{W}}e^{-2\rho}W_{-2} + e^{\rho}T_1^+ - \frac{2\pi}{k}\bar{\mathcal{L}}_2e^{-\rho}T_{-1}^+ \right) dx^+ \\ & - \bar{\mu} \left(e^{2\rho}W_2 - \frac{4\pi\bar{\mathcal{L}}_1}{k}W_0 + \frac{4\pi^2\bar{\mathcal{L}}_1^2}{k^2}e^{-2\rho}W_{-2} + \frac{4\pi\bar{\mathcal{W}}}{k}e^{-\rho}T_{-1}^- \right) dx^- + 2\xi Jd\phi, \\ & - (T_0^- + T_0^+)d\rho. \end{aligned} \quad (3.86)$$

with $\sigma = (3/4)^2$. The metric due to the above gauge connections is

$$\begin{aligned} ds^2 = & d\rho^2 - 3 \left(\mu e^{2\rho}dx^- + \frac{16\pi}{18k}\bar{\mathcal{W}} + \frac{4\pi^2}{k^2}\bar{\mu}\bar{\mathcal{L}}_1^2e^{-2\rho}dx^+ \right) \\ & \times \left(\bar{\mu}e^{2\rho}dx^- + \frac{16\pi}{18k}\mathcal{W} + \frac{4\pi^2}{k^2}\mu\mathcal{L}_1^2e^{-2\rho}dx^+ \right) \\ & - \frac{4}{3} \left(e^{\rho}dx^+ - \frac{2\pi}{k}\bar{\mathcal{L}}_1e^{\rho}dx^- + \frac{4\pi}{k}\bar{\mu}\bar{\mathcal{W}}e^{-\rho}dx^+ \right) \left(e^{\rho}dx^+ - \frac{2\pi}{k}\mathcal{L}_1e^{\rho}dx^- + \frac{4\pi}{k}\mu\mathcal{W}e^{-\rho}dx^+ \right) \\ & - \frac{1}{4} \left(\frac{4\pi}{k} \right)^2 (\mu\mathcal{L}_1dx^- + \bar{\mu}\mathcal{L}_1dx^+)^2 - \frac{2\pi}{3k} (\mathcal{L}_2(dx^+)^2 + \bar{\mathcal{L}}_2(dx^-)^2) \\ & + \frac{1}{3} \left(e^{2\rho} + \left(\frac{2\pi}{k} \right)^2 \mathcal{L}_2\bar{\mathcal{L}}_2e^{-2\rho} \right) dx^+dx^- \end{aligned} \quad (3.87)$$

We shall again consider the supersymmetry of the black hole with $\mathcal{W} = 0$ and $\mu = 0$. The equation for the Killing spinor is given by

$$\mathcal{D}_{\mu}\lambda \equiv \partial_{\mu}\lambda + [A_{\mu}, \lambda] = 0, \quad (3.88)$$

where λ is given by

$$\lambda \equiv \sum_{r=-1/2}^{1/2} \epsilon^r G_r^+ + \sum_{r=-1/2}^{1/2} \tilde{\epsilon}^r G_r^- + \sum_{r=-3/2}^{3/2} \lambda^r U_r^+ + \sum_{r=-3/2}^{3/2} \tilde{\lambda}^r U_r^-. \quad (3.89)$$

Written in matrix form the equation in (3.88) reads

$$\partial_\rho \begin{pmatrix} \epsilon^{-1/2} \\ \epsilon^{1/2} \\ \lambda^{-3/2} \\ \lambda^{-1/2} \\ \lambda^{1/2} \\ \lambda^{3/2} \end{pmatrix} + \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{3}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{3}{2} \end{pmatrix} \begin{pmatrix} \epsilon^{-1/2} \\ \epsilon^{1/2} \\ \lambda^{-3/2} \\ \lambda^{-1/2} \\ \lambda^{1/2} \\ \lambda^{3/2} \end{pmatrix} = 0, \quad (3.90)$$

$$\partial_+ \begin{pmatrix} \epsilon^{-1/2} \\ \epsilon^{1/2} \\ \lambda^{-3/2} \\ \lambda^{-1/2} \\ \lambda^{1/2} \\ \lambda^{3/2} \end{pmatrix} + \begin{pmatrix} \xi & \frac{2\pi(4\mathcal{L}_1-\mathcal{L}_2)}{3c} & 0 & 0 & -\frac{2i\pi(\mathcal{L}_1-\mathcal{L}_2)}{3c} & 0 \\ 1 & \xi & 0 & 0 & 0 & -\frac{2i\pi(\mathcal{L}_1-\mathcal{L}_2)}{c} \\ -\frac{4i\pi(\mathcal{L}_1-\mathcal{L}_2)}{3c} & 0 & \xi & \frac{2\pi(2\mathcal{L}_1+\mathcal{L}_2)}{3c} & 0 & 0 \\ 0 & -\frac{4i\pi(\mathcal{L}_1-\mathcal{L}_2)}{3c} & 3 & \xi & \frac{4\pi(2\mathcal{L}_1+\mathcal{L}_2)}{3c} & 0 \\ 0 & 0 & 0 & 2 & \xi & \frac{2\pi(2\mathcal{L}_1+\mathcal{L}_2)}{c} \\ 0 & 0 & 0 & 0 & 1 & \xi \end{pmatrix} \begin{pmatrix} \epsilon^{-1/2} \\ \epsilon^{1/2} \\ \lambda^{-3/2} \\ \lambda^{-1/2} \\ \lambda^{1/2} \\ \lambda^{3/2} \end{pmatrix} = 0, \quad (3.91)$$

$$\partial_- \begin{pmatrix} \epsilon^{-1/2} \\ \epsilon^{1/2} \\ \lambda^{-3/2} \\ \lambda^{-1/2} \\ \lambda^{1/2} \\ \lambda^{3/2} \end{pmatrix} + \begin{pmatrix} -\xi & 0 & 0 & 0 & 0 & 0 \\ 0 & -\xi & 0 & 0 & 0 & 0 \\ 0 & 0 & -\xi & 0 & 0 & 0 \\ 0 & 0 & 0 & -\xi & 0 & 0 \\ 0 & 0 & 0 & 0 & -\xi & 0 \\ 0 & 0 & 0 & 0 & 0 & -\xi \end{pmatrix} \begin{pmatrix} \epsilon^{-1/2} \\ \epsilon^{1/2} \\ \lambda^{-3/2} \\ \lambda^{-1/2} \\ \lambda^{1/2} \\ \lambda^{3/2} \end{pmatrix} = 0. \quad (3.92)$$

The solutions to these equations are given by

$$\begin{pmatrix} \epsilon^{-1/2} \\ \epsilon^{1/2} \\ \lambda^{-3/2} \\ \lambda^{-1/2} \\ \lambda^{1/2} \\ \lambda^{3/2} \end{pmatrix} = \mathcal{R}(\rho) f_+(x_+) f_-(x_-), \quad (3.93)$$

where, $\mathcal{R}(\rho)$ is the square matrix in (3.6) which contains the ρ -dependence. The x_+ and x_- dependent pieces are as follows

$$\begin{aligned}
f_+(x_+) = & c_1 e^{-\left(\sqrt{\frac{2\pi\mathcal{L}_2}{k}} + \xi\right)x_+} \mathbf{y}_1 + c_2 e^{-\left(\sqrt{\frac{2\pi\mathcal{L}_2}{k}} + \xi\right)x_+} \mathbf{y}_2 + c_3 e^{-\left(-\sqrt{\frac{2\pi\mathcal{L}_2}{k}} + 2\sqrt{\frac{2\pi\mathcal{L}_1}{k}} + \xi\right)x_+} \mathbf{y}_3 \\
& + c_4 e^{-\left(\sqrt{\frac{2\pi\mathcal{L}_2}{k}} - 2\sqrt{\frac{2\pi\mathcal{L}_1}{k}} + \xi\right)x_+} \mathbf{y}_4 + c_5 e^{-\left(-\sqrt{\frac{2\pi\mathcal{L}_2}{k}} - 2\sqrt{\frac{2\pi\mathcal{L}_1}{k}} + \xi\right)x_+} \mathbf{y}_5 \\
& + c_6 e^{-\left(\sqrt{\frac{2\pi\mathcal{L}_2}{k}} + 2\sqrt{\frac{2\pi\mathcal{L}_1}{k}} + \xi\right)x_+} \mathbf{y}_6,
\end{aligned} \tag{3.94}$$

\mathbf{y}_i are the eigenvectors of the matrix that appears in the $+$ component of the Killing spinor equation.

$$f_-(x_-) = e^{\xi x_-}. \tag{3.95}$$

The value of the $u(1)$ field for which we obtain periodic Killing spinors is given by

$$\xi = \pm \left(\sqrt{\frac{2\pi\mathcal{L}_1}{k}} \pm \frac{1}{2} \sqrt{\frac{2\pi\mathcal{L}_2}{k}} \right) \quad \text{or,} \quad \xi = \pm \frac{1}{2} \sqrt{\frac{2\pi\mathcal{L}_2}{k}}. \tag{3.96}$$

Thus generically the solution admits a single Killing spinor.

For the case of the black holes in this paper we have explicitly solved the Killing spinor components of G_r^+ and U_r^+ . The same method can be employed to solve for the components of the G_r^- and U_r^- generators as well.

3.5 Summary of the solutions and their supersymmetry

We now summarize the results for the Killing spinors for the various classical solutions of the $sl(3|2)$ Chern-Simons theory considered in this paper. The generic number of Killing spinors and their periodicity condition listed in this table correspond to the G^+ and U^- generators of the theory.

Table 1

Background	Killing spinor condition	Number of Killing spinors
BTZ black hole in $sl(2)$ with $M = J$	Periodic Killing spinors	3
BTZ black hole with $M = J$	Periodic Killing spinors	2
Higher spin black hole of Gutperle et al. with $\mathcal{W} = \mu = 0$	$\xi = \pm \sqrt{\frac{2\pi\mathcal{L}}{k}}$ or $\xi = i\frac{n}{2}$	2
New R-charged higher spin black hole with $\mathcal{W} = \mu = 0$	$\xi = \pm \left(\sqrt{\frac{2\pi\mathcal{L}_1}{k}} \pm \frac{1}{2} \sqrt{\frac{2\pi\mathcal{L}_2}{k}} \right)$ or $\xi = \pm \frac{1}{2} \sqrt{\frac{2\pi\mathcal{L}_2}{k}}$	1
AdS_3 in $sl(2)$	Anti-periodic Killing spinors	6
AdS_3	Anti-periodic Killing spinors	6
Higher spin conical defect	$2\xi = \pm i\sqrt{\alpha}\delta + in$ $2\xi = \pm i\sqrt{\alpha} \left(\delta \pm 2(\beta^2 - (\frac{3}{4}\eta)^2)^{1/2} \right) + in$	1
Conical defects in $sl(2)$	$\xi = \pm i\frac{\sqrt{\gamma}}{4} + in$	3
Conical defects in gravitational $sl(2)$	$\xi = \pm i\frac{\sqrt{\gamma}}{4} + in$ or $\xi = \pm 3i\frac{\sqrt{\gamma}}{4} + in.$	1

4. Supersymmetry and holonomy

In the previous section we have solved for the conditions under which the background solution admits periodic Killing spinors. This was a tedious but straight forward exercise. Since the Chern-Simons action is independent of the metric on the manifold it must be possible to state these conditions in terms of gauge independent and well defined physical observables. In this section we show that the periodicity conditions for the Killing spinor can be written in terms of a condition on the eigenvalues of holonomy of the background gauge connection around the angular ϕ direction. This invariant characterization of supersymmetry in higher spin theories in 3 dimension is the central result of this work. We state the condition for a general gauge connection belonging to the $sl(N|N-1)$ superalgebra. We show that whenever the holonomy of the $u(1)$ part of the connection along with eigenvalues of the rest of the background holonomy weighted with the odd roots of the superalgebra becomes trivial then the Killing spinor is periodic. This condition is given in equation (4.9). We then explicitly verify that this condition reproduces the equations (3.30) and (3.31) we find for the higher spin conical defects in the $sl(3|2)$ algebra. We have also verified that the holonomy condition reproduces the periodicity of Killing spinor for the black holes considered in the $sl(3|2)$ theory. We then proceed to combine the supersymmetry requirement along with the requirement that the holonomy is smooth to show that for $N \geq 4$ the $sl(N|N-1)$ theory admits smooth and supersymmetric conical defects.

4.1 Killing spinor periodicity as a holonomy

The equation for the covariantly constant Killing spinor satisfies the equation given by

$$\mathcal{D}_\mu \epsilon \equiv \partial_\mu \epsilon + [A_\mu, \epsilon] = 0. \quad (4.1)$$

Here $\epsilon = \epsilon^i G_i$ is a linear combination of the fermionic generators. $A_\mu = A_\mu^a T_a$ are the connection one forms valued in the bosonic part of the algebra. It is convenient to choose the fermionic generators in the Cartan-Weyl basis of the super algebra. For definiteness we can work with the super group $sl(N|N-1)$ but the discussion can be easily generalized to any super algebra. In the Cartan-Weyl basis, the generators satisfy the following conditions: let H_r be the Cartan's of the superalgebra and J be the $U(1)$. Then we have the commutation relations

$$[H_r, G_i] = \alpha_i^r G_i, \quad [J, G_i] = \pm G_i, \quad (4.2)$$

where α_i^r is the r th component of the odd root α_i . As mentioned in section 2 we see that the integrability condition for the Killing spinor equation is satisfied since the background gauge field satisfies the equation of motion. We can therefore solve the equation in (4.1) formally by writing the solution as

$$\epsilon(x) = \mathcal{P}(e^{\int_{x_0}^x A_\mu dx^\mu}) \hat{\epsilon}(x_0) \mathcal{P}(e^{-\int_{x_0}^x A_\mu dx^\mu}), \quad (4.3)$$

where x_0 is a base point and $\hat{\epsilon}(x_0)$ is a constant spinor and \mathcal{P} refers to the path ordered exponential. To determine whether the spinor is periodic we can consider $x = (\rho, t, 2\pi)$ and $x_0 = (\rho, t, 0)$ and the integral is along the constant time circle in the angular direction. For all the solutions considered in this paper, the holonomy along this circle reduces to the form

$$\text{Hol}_\phi(A) = \mathcal{P} \exp\left(\oint A_\mu dx^\mu\right) = b^{-1}(\rho) \exp\left(\oint a_\phi d\phi\right) b(\rho), \quad (4.4)$$

where $b(\rho)$ is the matrix which contains the ρ dependence. The connection a_ϕ is constant and can be easily integrated. Since it is a sum of the bosonic generators we can write it as

$$\exp\left(\oint a_\phi d\phi\right) = S^{-1} \exp(2\pi(\lambda^r H_r + 2\xi J)) S, \quad (4.5)$$

where S is the similarity transformation which brings the constant holonomy in the diagonal form. Now substituting the equation (4.5) in the solution of the Killing spinor given in (4.3) we find the periodicity of the spinor is determined by

$$\epsilon(\rho, t, 2\pi) = b^{-1} S^{-1} e^{2\pi(\lambda^r H_r + \xi J)} S b(\rho) \hat{\epsilon}(\rho, t, 0) b^{-1}(\rho) S^{-1} e^{-2\pi(\lambda^r H_r + 2\xi J)} S b. \quad (4.6)$$

Since the Cartan-Weyl basis for fermionic generators G_i is complete we have the relation

$$S b(\rho) \hat{\epsilon}(\rho, t, 0) b^{-1}(\rho) S^{-1} = \epsilon(\rho, t, 0) = \tilde{\epsilon}^i(\rho, t, 0) G_i. \quad (4.7)$$

From the commutation relations given in (4.2) we find

$$e^{2\pi(\lambda^r H_r + \xi J)} G^i e^{-2\pi(\lambda^r H_r + 2\xi J)} = e^{2\pi(\lambda^r \alpha_i^r \pm 2\xi)} G_i. \quad (4.8)$$

Now substitute equations (4.7) and (4.8) into the periodicity constraint for the Killing spinor given in (4.6). Let us consider the case in which say any one of the $\tilde{\epsilon}^i$ is turned on and the rest set to zero. Then we see that the spinor with $\tilde{\epsilon}^i$ along the generator G^i is periodic if the following condition is true

$$\lambda^r \alpha_i^r \pm 2\xi = in. \quad (4.9)$$

where n is any integer and r is summed over the Cartan directions other than the $U(1)$. Recall that λ^r are the eigenvalues of the holonomy of the background connection, α_i^r are the odd roots of the Cartan generator H_r and ξ is the value of the $U(1)$ field. Note that the sign \pm depends on the sign of the commutation relation $[J, G^i] = \pm G^i$. Thus we find that the periodicity property of the Killing spinor along the ϕ direction can be generally stated in terms of product of the eigenvalues of the holonomies of the background connection with the odd roots of the super algebra. The number of supersymmetries preserved can also be found easily by checking how many among all the fermionic directions labelled by i satisfy the condition (4.9). We have thus shown that the supersymmetry condition on any background can be written in terms of gauge invariant and physically independent observables.

A test of the supersymmetry condition

We will now verify the general equation for the periodicity of the Killing spinor derived in (4.9) for the specific situation of higher spin conical defects in the $sl(3|2)$ theory. From the gauge connection in (3.12) we obtain

$$a_\phi = \delta_{-1} T_{-1}^+ + \delta_1 T_1^+ + \beta_{-1} T_{-1}^- + \beta_1 T_1^- + \eta_{-1} W_{-1} + \eta_1 W_1 + 2\xi J. \quad (4.10)$$

We now use representation of the matrices for $sl(3)$ given in [18] with $\sigma = (\frac{3}{4})^2$ and the following representation for $sl(2)$ in terms of the Pauli matrices

$$T_1^+ = \frac{1}{2}(\sigma_1 - i\sigma_2), \quad T_{-1}^+ = \frac{1}{2}(\sigma_1 + i\sigma_2), \quad T_0^+ = \frac{1}{2}\sigma_3. \quad (4.11)$$

Then the eigenvalues of the $sl(3) \oplus sl(2)$ part of the matrix a_ϕ along with the $u(1)$ part is given by

$$Sa_\phi S^{-1} = \text{Diag} \left[2i\sqrt{\alpha(\beta^2 - (\frac{3}{4}\eta)^2)}, 0, -2i\sqrt{\alpha(\beta^2 - (\frac{3}{4}\eta)^2)}, i\sqrt{\alpha}\delta, -i\sqrt{\alpha}\delta \right] + 2\xi J. \quad (4.12)$$

We will now write this as a linear combination of the Cartan generators of $sl(3|2)$. From the appendix which lists the generators of $sl(N|N-1)$, we find that (4.12) can be written as

$$Sa_\phi S^{-1} = 2i\sqrt{\alpha(\beta^2 - (\frac{3}{4}\eta)^2)}(H_1 + H_2) + i\sqrt{\alpha}\delta H_4 + 2\xi J, \quad (4.13)$$

where the Cartan matrices are given by

$$H_1 = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & 0 & \\ & & & 0 \end{pmatrix} \quad H_2 = \begin{pmatrix} 0 & & & \\ & 1 & & \\ & & -1 & \\ & & & 0 \end{pmatrix} \quad H_{\bar{4}} = \begin{pmatrix} 0 & & & \\ & 0 & & \\ & & 0 & \\ & & & 1 \\ & & & & -1 \end{pmatrix}. \quad (4.14)$$

In this representation the $U(1)$ generator J is given by

$$J = \begin{pmatrix} -2 & & & & \\ & -2 & & & \\ & & -2 & & \\ & & & -3 & \\ & & & & -3 \end{pmatrix}. \quad (4.15)$$

We now need the odd roots of the supercharges with J charge 1. In the Cartan-Weyl basis these are given by 6 matrices $E_{\bar{i},k}$ with $\bar{i} = \bar{4}, \bar{5}$ and $k = 1, 2, 3$. They correspond to the 6 generators $G_{\pm 1/2}^+, U_{\pm 1/2}^+, U_{\pm 3/2}^+$. Evaluating the commutation relations explicitly using the matrix representation given in the appendix we find the following roots

$$\begin{aligned} [H_1 + H_2, E_{\bar{4}1}] &= -E_{\bar{4}1}, & [H_1 + H_2, E_{\bar{4}2}] &= 0, & [H_1 + H_2, E_{\bar{4}3}] &= E_{\bar{4}3}, \\ [H_{\bar{4}}, E_{\bar{4}k}] &= E_{\bar{4}k}, & [H_{\bar{4}}, E_{\bar{5}k}] &= -E_{\bar{5}k}. \end{aligned} \quad (4.16)$$

We now have all the information to derive the supersymmetric conditions given in (4.9). Consider the supercharge $E_{\bar{4}1}$, using the holonomy of the background given in (4.12) and the roots from (4.16) we find the following condition

$$-2i\sqrt{\alpha(\beta^2 - (\frac{3}{4}\eta)^2)} + i\sqrt{\alpha}\delta + 2\xi = in. \quad (4.17)$$

We see that this matches with one of equations in (3.31). Now consider the supercharge $E_{\bar{4}2}$, again using (4.12) and (4.16) we obtain

$$i\sqrt{\alpha}\delta + 2\xi = in. \quad (4.18)$$

This coincides with one of the equations in (3.30). Repeating this explicitly for all the remaining supercharges we obtain the 6 conditions in (3.30) and (3.31). We have also verified that the supersymmetry condition (4.9) reproduces the conditions for the periodicity of the Killing spinors for the case of black holes in the $sl(3|2)$ theory studied in this paper.

4.2 Smooth holonomy and supersymmetry

Smooth conical defects have played a central role in tests of the minimal model/higher spin duality. They are dual to the primaries of the \mathcal{W}_N minimal model after a suitable analytical continuation [19]. A Kazama-Suzuki supersymmetric minimal model was proposed to be dual to the large N limit of the $sl(N|N-1)$ higher spin theories studied in this paper [14]. Thus we expect smooth conical defects to be dual to primaries of the supersymmetric minimal model. However in a supersymmetric theory there are special primaries called chiral primaries which preserve supersymmetry. They are protected against quantum corrections and can be used as probes to test the minimal model/higher spin duality. Thus smooth supersymmetric conical defects of the $sl(N|N-1)$ theory are expected to be dual to the chiral primaries of the Kazama-Suzuki minimal model after an analytic continuation to infinite N or finite central charge ¹. Note that all the conical defects considered in this paper are metrically singular as seen from the metric written in equation (3.23). However since the circle around the angular direction ϕ , is contractable a gauge invariant method to decide when the solution is smooth is to consider the holonomy of the Chern-Simons connection around this circle [19]. A solution is smooth if this holonomy is trivial. With this motivation we study the conditions under which a conical defect is both smooth as well as supersymmetric. We first begin with the $sl(3|2)$ theory and show that it does not admit smooth supersymmetric conical defects. The supersymmetric defects are singular in this case, that is they do not admit a smooth holonomy. We then study supersymmetry and smoothness for conical defects in $sl(N|N-1)$ theories for $N \geq 4$. For these theories it is shown that there are smooth and supersymmetric conical defects.

Smoothness and supersymmetry: $sl(3|2)$

Let us now focus on the the $sl(3|2)$ theory and investigate if the theory admits smooth supersymmetric conical defects. As discussed above we first demand that the holonomy around the angular direction is trivial. This leads to the following conditions on δ , β and η .

$$\begin{aligned}\sqrt{\alpha}\delta &= \pm \frac{m}{2}, \\ 2\sqrt{\alpha}(\beta^2 - (\frac{3}{4}\eta)^2)^{1/2} &= \pm p, \\ -2i\xi &= \pm q.\end{aligned}\tag{4.19}$$

where m , p , $q \in \mathbb{Z}$. Note that the values of $\sqrt{\alpha}\delta$ are quantized in half integers because the center of $SL(2)$ is \mathbb{Z}_2 valued. Substituting the periodicity conditions of

¹The gravitational higher spin theory we are studying is classical and therefore has large central charge

the Killing spinors given in (3.30) and (3.31) we obtain the following conditions

$$q \mp \frac{m}{2} = n, \quad q \mp \left(\frac{m}{2} \pm p\right) = n. \quad (4.20)$$

Let us now examine if any of the conical defects in the $sl(3|2)$ theory satisfy the requirement that they lie in the domain pointed out in [19]

$$-\frac{c}{24} < L_0 < 0, \quad (4.21)$$

where c is the central charge of the theory which can be written in terms of the cosmological constant. This restriction comes from the fact that we need these solutions to have mass above the AdS_3 vacuum and below the zero mass BTZ. For the $sl(3|2)$ theory the value L_0 in terms of the holonomy is given by

$$L_0 = \frac{c}{24\epsilon_{(3|2)}} \left(\text{str}(a_\phi^2)\right) \quad (4.22)$$

Note that by defining L_0 as given in (4.22) the shift in energy due to the presence of the presence of the $U(1)$ field [36, 27] is accounted for. Substituting the values of the holonomy from (4.19) in (4.22) we obtain the following bound that the integers p, q, m must satisfy

$$0 < p^2 - \left(\frac{m}{2}\right)^2 - 6q^2 < \frac{3}{4}. \quad (4.23)$$

The factor of 6 occurs on taking the super trace of J^2 . This can be seen by using the definition of J given in (4.15). On substituting for $m/2$ from the supersymmetric holonomy conditions given in (4.20) we obtain the following bounds

$$0 < p^2 - (q - n)^2 - 6q^2 < 3/4, \quad 0 < p^2 - (q - n \mp p)^2 - 6q^2 < 3/4. \quad (4.24)$$

It is clear that any of the above bounds are satisfied since there is no integer between 0 and 3/4. Thus there are no supersymmetric smooth conical defects in the $sl(3|2)$.

Smoothness and supersymmetry: $sl(N|N - 1)$, $N \geq 5$

We will now look at the $sl(N|N - 1)$ Chern-Simons theory for $N \geq 5$ and show that the theory admits smooth conical defects. The case of $N = 4$ will be treated later, the reason is that for $N \geq 5$, the Cartan generators of $sl(N|N - 1)$ can be stated in a simple form. We shall be using the algebra and the matrix representation of the generators given in Section 61 of [34]. We have reviewed this representation in Appendix A. Following [19] we can write the gauge connections for the conical defects in the $sl(N|N - 1) \oplus sl(N|N - 1)$ theory as

$$A = b^{-1}ab + b^{-1}db, \quad \bar{A} = b^{-1}\bar{a}b + b^{-1}d\bar{b}, \quad (4.25)$$

where $b = \exp(\rho \hat{L}_0)$. \hat{L}_0 is the $sl(2)$ generator which is principally embedded in $sl(N|N-1)$. Explicitly it is given by the diagonal $(2N-1) \times (2N-1)$ matrix whose diagonal elements are given by

$$2\hat{L}_0 = \text{Diag}(N-1, N-3, \dots, -(N-3), -(N-1), \\ N-2, N-4, \dots, -(N-4), -(N-2)). \quad (4.26)$$

The connections a and \bar{a} are given by

$$a = \left(\sum_{k=1}^{N-1} B_k(a_k, \alpha a_k) + \sum_{\bar{k}=\overline{N+1}}^{\overline{2N-2}} B_{\bar{k}}(a_{\bar{k}}, \alpha a_{\bar{k}}) \right) dx^+ + 2\xi J, \\ \bar{a} = - \left(\sum_{k=1}^{N-1} B_k(\gamma a_k, \frac{\gamma}{\alpha} a_k) + \sum_{\bar{k}=\overline{N+1}}^{\overline{2N-2}} B_{\bar{k}}(\gamma a_{\bar{k}}, \frac{\gamma}{\alpha} a_{\bar{k}}) \right) dx^- + 2\xi J, \quad (4.27)$$

where ‘ B -matrices’ are defined as

$$[B_K(x, y)]_{IJ} = x\delta_{I,K}\delta_{J,K+1} - y\delta_{I,K+1}\delta_{J,K}. \quad (4.28)$$

I, J and K values run from $1, 2, \dots, N, \overline{N+1}, \overline{N+2}, \dots, \overline{2N-1}$. The connection given in (4.27) contains the conical defects found by [19] in both the algebras $sl(N)$ and $sl(N-1)$ together with the $u(1)$ field turned on. We now have to diagonalize the connection a_ϕ to find the eigenvalues of the holonomy matrix. When the connection given in (4.27) is brought to the diagonal form, it can be written as a linear combination of the Cartan generators H_J of $sl(N|N-1)$ given in the Appendix. For $N \geq 5$ we obtain

$$S a_\phi S^{-1} = \begin{cases} i \sum_{\substack{j=1 \\ j \text{ odd}}}^{N-1} a_j H_j + i \sum_{\substack{j=\overline{N+1} \\ j \text{ odd}}}^{\overline{2N-3}} a_j H_j + 2\xi J, & \text{for } N \text{ even.} \\ i \sum_{\substack{j=1 \\ j \text{ odd}}}^{N-2} a_j H_j + i \sum_{\substack{j=\overline{N+1} \\ j \text{ even}}}^{\overline{2N-2}} a_j H_j + 2\xi J, & \text{for } N \text{ odd.} \end{cases} \quad (4.29)$$

On imposing trivial holonomies for smoothness of the conical defect solutions, we have

$$\text{For even } N \quad : a_i = \frac{m_i}{2}, \quad a_{\bar{i}} = p_{\bar{i}}, \quad -2i\xi = q, \\ \text{For odd } N \quad : a_i = r_i, \quad a_{\bar{i}} = \frac{s_{\bar{i}}}{2}, \quad -2i\xi = t. \quad (4.30)$$

Here $p_{\bar{i}}, q, r_i, t \in \mathbb{Z}$ and m_i and $s_{\bar{i}}$ take values in the set of either even or odd integers for all i and \bar{i} respectively. The reason that for even N , a_i takes values in the set of half integers is because the group $SL(N)$ has a \mathbb{Z}_2 valued center. Similarly for odd N the group $SL(N-1)$ has a \mathbb{Z}_2 valued center which makes $a_{\bar{i}}$ takes values in the set of half integers.

The next step is to find out the roots of the fermionic generators, $E_{i\bar{j}}$ with the linear combination of the Cartan matrices given in (4.29). A generic linear combination can be written as $\sum_k a_k H_k + \sum_{\bar{l}} a_{\bar{l}} H_{\bar{l}}$. The commutator of this with a fermionic generator is

$$\left[i \sum_k a_k H_k + i \sum_{\bar{l}} a_{\bar{l}} H_{\bar{l}}, E_{i\bar{j}} \right] = i[(a_i - a_{i-1}) - (a_{\bar{j}} - a_{\bar{j}-1})] E_{i\bar{j}}. \quad (4.31)$$

Here $a_0 = a_{\bar{0}} = 0$ and these fermionic generators have $u(1)$ charge $+1$. Using these roots we can write out the periodicity condition for the Killing spinors given in (4.9). We see that the conditions split to four cases each for even and odd N . For even N

$$\begin{aligned} \text{odd } i \text{ and odd } \bar{j} &: i(a_i - a_{\bar{j}}) + 2\xi = in_{i\bar{j}}, \\ \text{odd } i \text{ and even } \bar{j} &: i(a_i + a_{\bar{j}-1}) + 2\xi = in_{i\bar{j}}, \\ \text{even } i \text{ and odd } \bar{j} &: -i(a_{i-1} + a_{\bar{j}}) + 2\xi = in_{i\bar{j}}, \\ \text{even } i \text{ and even } \bar{j} &: -i(a_{i-1} - a_{\bar{j}-1}) + 2\xi = in_{i\bar{j}}. \end{aligned} \quad (4.32)$$

Whereas for odd N we have

$$\begin{aligned} \text{odd } i \text{ and odd } \bar{j} &: i(a_i + a_{\bar{j}-1}) + 2\xi = in_{i\bar{j}}, \\ \text{odd } i \text{ and even } \bar{j} &: i(a_i - a_{\bar{j}}) + 2\xi = in_{i\bar{j}}, \\ \text{even } i \text{ and odd } \bar{j} &: -i(a_{i-1} - a_{\bar{j}-1}) + 2\xi = in_{i\bar{j}}, \\ \text{even } i \text{ and even } \bar{j} &: -i(a_{i-1} + a_{\bar{j}}) + 2\xi = in_{i\bar{j}}. \end{aligned} \quad (4.33)$$

where $n_{i\bar{j}} \in \mathbb{Z}$. Thus there are $N(N-1)$ such conditions which is equal to the number of fermionic generators with positive $u(1)$ charge. Substituting the quantization conditions of (4.30) in the above equations we obtain the following constraints from the periodicity of the Killing spinors.

$$\text{even } N : \begin{cases} \text{odd } i \text{ and odd } \bar{j} & : \frac{m_i}{2} - p_{\bar{j}} + q = n_{i\bar{j}}, \\ \text{odd } i \text{ and even } \bar{j} & : \frac{m_i}{2} - p_{\bar{j}-1} + q = n_{i\bar{j}}, \\ \text{even } i \text{ and odd } \bar{j} & : -\frac{m_{i-1}}{2} - p_{\bar{j}} + q = n_{i\bar{j}}, \\ \text{even } i \text{ and even } \bar{j} & : -\frac{m_{i-1}}{2} + p_{\bar{j}-1} + q = n_{i\bar{j}}. \end{cases} \quad (4.34)$$

$$\text{odd } N : \begin{cases} \text{odd } i \text{ and odd } \bar{j} & : r_i - \frac{s_{\bar{j}-1}}{2} + t = n_{i\bar{j}}, \\ \text{odd } i \text{ and even } \bar{j} & : r_i - \frac{s_{\bar{j}}}{2} + t = n_{i\bar{j}}, \\ \text{even } i \text{ and odd } \bar{j} & : -r_{i-1} + \frac{s_{\bar{j}-1}}{2} + t = n_{i\bar{j}}, \\ \text{even } i \text{ and even } \bar{j} & : -r_{i-1} - \frac{s_{\bar{j}}}{2} + t = n_{i\bar{j}}. \end{cases} \quad (4.35)$$

Finally we have to impose the bound (4.21) on the gauge connection for the conical defect. For the $sl(N|N-1)$ case, the charge L_0 in terms of the holonomy of

the background is given by

$$L_0 = \frac{c}{24\epsilon_{(N|N-1)}} (\text{str}(a_\phi^2)), \quad (4.36)$$

and

$$\epsilon_{(N|N-1)} = \text{str}(\hat{L}_0 \hat{L}_0) = \frac{1}{4}N(N-1). \quad (4.37)$$

Here we have used the explicit representation of \hat{L}_0 given in (4.26). Then equation (4.21) reduces to

$$0 < \sum_k a_k^2 - \sum_{\bar{l}} a_{\bar{l}}^2 + 4N(N-1)\xi^2 < \frac{N(N-1)}{8}. \quad (4.38)$$

Now substituting the quantization conditions given in (4.30) for even and odd N the bound can be written as

$$\text{For even } N : 0 < \frac{1}{4} \sum_{\substack{j=1 \\ j \text{ odd}}}^{N-1} m_j^2 - \sum_{\substack{\bar{j}=N+1 \\ j \text{ odd}}}^{\frac{2N-3}{2}} p_{\bar{j}}^2 - N(N-1)q^2 < \frac{N(N-1)}{8}, \quad (4.39)$$

$$\text{For odd } N : 0 < \sum_{\substack{j=1 \\ j \text{ odd}}}^{N-1} r_j^2 - \frac{1}{4} \sum_{\substack{\bar{j}=N+1 \\ j \text{ even}}}^{\frac{2N-2}{2}} s_{\bar{j}}^2 - N(N-1)t^2 < \frac{N(N-1)}{8}.$$

The theory admits smooth as well as supersymmetric conical defects if the above bound together with the periodicity conditions in (4.34) and (4.35) are satisfied. We will now provide some simple examples to demonstrate that smooth supersymmetric conical defects are allowed for $N \geq 5$. For the $N = 5$ case the bound reduces to

$$0 < (r_1^2 + r_3^2) - \frac{1}{4}(s_6^2 + s_8^2) - 20t^2 < \frac{5}{2}. \quad (4.40)$$

This inequality is satisfied for $r_1 = r_3 = 1$, $s_6 = s_8 = 2$, $t = 0$. While for $N = 6$ we have

$$0 < \frac{1}{4}(m_1^2 + m_3^2 + m_5^2) - (p_7^2 + p_9^2) - 30q^2 < \frac{15}{4}. \quad (4.41)$$

This inequality is satisfied for $m_1 = m_3 = m_5 = 2$, $p_7 = p_9 = 1$, $q = 0$. It is clear from these examples that as N gets larger the term on the extreme RHS of the inequalities in (4.39) increases and it should be possible to find more integers $m_j, p_{\bar{j}}, r_j, s_{\bar{j}}, q, t$ to satisfy the inequality.

Smoothness and supersymmetry: $sl(4|3)$

As mentioned earlier the case for $N = 4$ needs to be treated separately since the form obtained by diagonalizing the connection given in (4.27) can not be written in

the general form given in (4.29). Diagonalizing the background connection for $N = 4$ and writing it as a linear combination of the Cartan generators we obtain

$$Sa_\phi S^{-1} = ia_1 H_1 + ia_3 H_3 + ia_{\bar{5}}(H_{\bar{5}} + H_{\bar{6}}). \quad (4.42)$$

Imposing trivial holonomies for smoothness of the conical defect solutions, we get the following quantization conditions

$$a_i = \frac{m_i}{2}, \quad a_{\bar{5}} = p_{\bar{5}}, \quad -2i\xi = q. \quad (4.43)$$

Here $p_i, q, m_i \in \mathbb{Z}$ and m_i takes values in either the set of even or odd integers for all i . The commutator of $Sa_\phi S^{-1}$ with fermionic generators $E_{i\bar{j}}$ are as follows

$$i = 1, 3 \quad \begin{cases} [Sa_\phi S^{-1}, E_{i\bar{5}}] = i(a_i - a_{\bar{5}})E_{i\bar{5}}, \\ [Sa_\phi S^{-1}, E_{i\bar{6}}] = ia_i E_{i\bar{6}}, \end{cases} \quad (4.44)$$

$$i = 2, 4 \quad \begin{cases} [Sa_\phi S^{-1}, E_{i\bar{5}}] = -i(a_{i-1} + a_{\bar{5}})E_{i\bar{5}}, \\ [Sa_\phi S^{-1}, E_{i\bar{6}}] = -ia_{i-1} E_{i\bar{6}}. \end{cases} \quad (4.45)$$

Substituting these roots in the supersymmetry conditions (4.9) we obtain the following periodicity conditions

$$\begin{aligned} i(a_1 - a_{\bar{5}}) + 2\xi &= in_{1\bar{5}}, & ia_1 + 2\xi &= in_{1\bar{6}}, & i(a_3 - a_{\bar{5}}) + 2\xi &= in_{3\bar{5}}, & ia_3 + 2\xi &= in_{3\bar{6}}, \\ -i(a_1 + a_{\bar{5}}) + 2\xi &= in_{2\bar{5}}, & -ia_1 + 2\xi &= in_{2\bar{6}}, & -i(a_3 + a_{\bar{5}}) + 2\xi &= in_{4\bar{5}}, & -ia_3 + 2\xi &= in_{4\bar{6}}. \end{aligned} \quad (4.46)$$

The $a_i, a_{\bar{i}}$ and ξ are further constrained by (4.43).

Upon imposing the energy bound condition (4.21) and using the quantization conditions in (4.43) we obtain the following inequality

$$0 < \frac{1}{4}(m_1^2 + m_3^2) - p_{\bar{5}}^2 - 12q^2 < \frac{3}{2}, \quad (4.47)$$

A simple example in which this inequality is satisfied along with the constraints in (4.46) is

$$m_1 = 2, \quad m_3 = p_{\bar{5}} = q = 0. \quad (4.48)$$

Thus smooth supersymmetric conical defects therefore do exist in the $sl(4|3)$ theory.

5. Conclusions

The main result of this paper is the observation that the supersymmetry conditions of a given background for the $sl(N|N-1)$ theory can be written down in terms of products of the eigenvalues of the background holonomies with the odd roots of the super algebra. Thus the periodicity constraint on the Killing spinor can be formulated

in terms of gauge invariant and physically independent observables. This condition is given in (4.9). We have also constructed a class of conical defects and black holes in the $sl(3|2)$ theory. These solutions in general have fields in $sl(3) \oplus sl(2) \oplus u(1)$ directions turned on. We have obtained the periodicity properties for the Killing spinors in these backgrounds explicitly by solving the Killing spinor equations. A summary of the solutions and the supersymmetry conditions is given in table 1 of 3.5. These conditions can be seen to be in agreement with the general constraint (4.9). Though the analysis which resulted in the supersymmetry condition given in (4.9) was done in the $sl(N|N-1)$ as a concrete example, the condition (4.9) is general and can be applied to a Chern-Simons theories based on any super group.

We have shown that for $N \geq 4$, the $sl(N|N-1)$ admits smooth supersymmetric conical defects. Just as smooth conical defects in the bosonic $sl(N)$ theory are dual to the primaries of the \mathcal{W}_N minimal model, the smooth supersymmetric conical defects should be dual to the chiral primaries of the supersymmetric minimal model conjectured to be the large N limit of the the $sl(N|N-1)$ theories [14]. It will be interesting to classify the chiral primaries of these supersymmetric minimal models and compare them with the supersymmetric conical defects found in this paper [37]. Conical surplus solutions in the bosonic $sl(N, C)$ Chern-Simons theory have been shown to agree with the light states of the dual minimal model [19]. It will be interesting to see if the Euclidean supersymmetric version of the Chern-Simons theory studied in this paper admits conical surplus solutions and check if they are supersymmetric. One can then verify if they correspond to possible light states in the dual Kazama-Suzuki model of [14]².

The black holes we constructed in the $sl(3|2)$ theory have in addition to fields in $sl(3)$ also fields in the extra $sl(2)$ turned on. It will be interesting to study their smoothness/holonomy and the thermodynamic properties of these black hole solutions and obtain an expression for their partition function both from the bulk theory and the CFT.

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A. Cartan-Weyl basis for $sl(N|N-1)$

One can construct following [34] a basis of matrices for the $sl(N|N-1)$ algebra. Let's consider $(2N-1)^2$ matrices e_{IJ} of order $2N-1$ so that $(e_{IJ})_{KL} = \delta_{IL}\delta_{JK}$

²We thank Rajesh Gopakumar for discussions regarding this point

($I, J, K, L = 1, \dots, 2N - 1$) and define the $(2N - 1)^2 - 1$ generators

$$E_{ij} = e_{ij} - \delta_{ij}(e_{kk} + e_{\bar{k}\bar{k}}), \quad E_{i\bar{j}} = e_{i\bar{j}}, \quad (\text{A.1})$$

$$E_{\bar{i}j} = e_{\bar{i}j} + \delta_{\bar{i}j}(e_{kk} + e_{\bar{k}\bar{k}}), \quad E_{\bar{i}\bar{j}} = e_{\bar{i}\bar{j}}, \quad (\text{A.2})$$

where i, j, \dots run from 1 to N and \bar{i}, \bar{j}, \dots from $N + 1$ to $2N - 1$. The generators for the various subalgebras of $sl(3|2)$ are as follows

- $u(1) : J = E_{kk} = -E_{\bar{k}\bar{k}} = -((N - 1)e_{kk} + Ne_{\bar{k}\bar{k}})$.

For the above mentioned matrix representation we get

$$J = (-(N - 1))\mathbf{1}_{N \times N} \oplus (-N)\mathbf{1}_{(N-1) \times (N-1)}$$

It then follows that $\text{str}(J^2) = -N(N - 1)$.

- $sl(N) : E_{ij} - \frac{1}{N}\delta_{ij}Z$.
- $sl(N - 1) : E_{\bar{i}\bar{j}} + \frac{1}{N-1}\delta_{\bar{i}\bar{j}}Z$.
- $(\overline{N}, N - 1)$ representation of $sl(N) \oplus sl(N - 1) \oplus u(1) : E_{i\bar{j}}$.
- $(N, \overline{N - 1})$ representation of $sl(N) \oplus sl(N - 1) \oplus u(1) : E_{\bar{i}j}$.

In the Cartan-Weyl basis, the generators are given by

- Cartan subalgebra

$$H_i = E_{ii} - E_{i+1, i+1}, \quad \text{for } 1 \leq i \leq N - 1, \quad (\text{A.3})$$

$$H_{\bar{i}} = E_{\bar{i}\bar{i}} - E_{\bar{i}+1, \bar{i}+1}, \quad \text{for } N + 1 \leq i \leq 2N - 2, \quad (\text{A.4})$$

$$H_N = E_{NN} + E_{N+1, N+1}. \quad (\text{A.5})$$

- Raising operators

$$E_{ij} \text{ with } i < j \text{ for } sl(N), \quad E_{\bar{i}\bar{j}} \text{ with } \bar{i} < \bar{j} \text{ for } sl(N - 1), \quad E_{i\bar{j}} \text{ for the odd part} \quad (\text{A.6})$$

- Lowering operators

$$E_{ji} \text{ with } i < j \text{ for } sl(N), \quad E_{\bar{j}\bar{i}} \text{ with } \bar{i} < \bar{j} \text{ for } sl(N - 1), \quad E_{\bar{i}j} \text{ for the odd part} \quad (\text{A.7})$$

The commutation relations in this basis are

$$\begin{aligned} [H_I, H_J] &= 0, \\ [H_K, E_{IJ}] &= \delta_{IK}E_{KJ} - \delta_{I, K+1}E_{K+1, J} - \delta_{KJ}E_{IK} + \delta_{K+1, J}E_{I, K+1} \quad (K \neq N), \\ [H_N, E_{IJ}] &= \delta_{Im}E_{NJ} + \delta_{I, N+1}E_{N+1, J} - \delta_{NJ}E_{Im} - \delta_{N+1, J}E_{I, N+1}, \\ [E_{IJ}, E_{KL}] &= \delta_{JK}E_{IL} - \delta_{IL}E_{KJ} \quad \text{for } E_{IJ} \text{ and } E_{KL} \text{ even}, \\ [E_{IJ}, E_{KL}] &= \delta_{JK}E_{IL} - \delta_{IL}E_{KJ} \quad \text{for } E_{IJ} \text{ even and } E_{KL} \text{ odd}, \\ \{E_{IJ}, E_{KL}\} &= \delta_{JK}E_{IL} + \delta_{IL}E_{KJ} \quad \text{for } E_{IJ} \text{ and } E_{KL} \text{ odd}. \end{aligned} \quad (\text{A.8})$$

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